

**Heat and Mass Transfer: Fundamentals & Applications**  
**Fourth Edition**  
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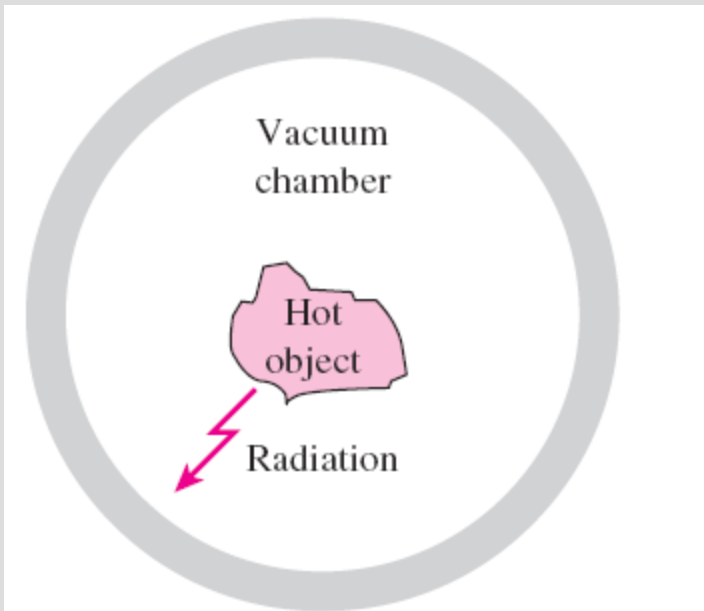
**Chapter 12**  
**FUNDAMENTALS OF THERMAL  
RADIATION**

# Objectives

- Classify electromagnetic radiation, and identify thermal radiation
- Understand the idealized blackbody, and calculate the total and spectral blackbody emissive power
- Calculate the fraction of radiation emitted in a specified wavelength band using the blackbody radiation functions
- Understand the concept of radiation intensity, and define spectral directional quantities using intensity
- Develop a clear understanding of the properties emissivity, absorptivity, reflectivity, and transmissivity on spectral and total basis
- Apply Kirchhoff law's to determine the absorptivity of a surface when its emissivity is known
- Model the atmospheric radiation by the use of an effective sky temperature, and appreciate the importance of greenhouse effect

# INTRODUCTION

The hot object in vacuum chamber will eventually cool down and reach thermal equilibrium with its surroundings by a heat transfer mechanism: **radiation**.

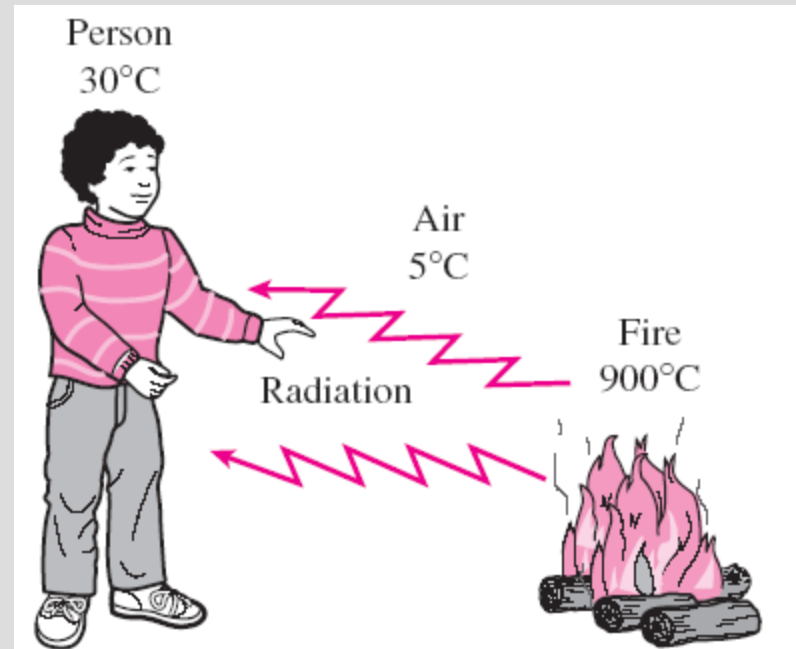


**FIGURE 12-1**

A hot object in a vacuum chamber loses heat by radiation only.

Radiation differs from conduction and convection in that it does not require the presence of a material medium to take place.

Radiation transfer occurs in solids as well as liquids and gases.



**FIGURE 12-2**

Unlike conduction and convection, heat transfer by radiation can occur between two bodies, even when they are separated by a medium colder than both.

Accelerated charges or changing electric currents give rise to electric and magnetic fields. These rapidly moving fields are called **electromagnetic waves** or **electromagnetic radiation**, and they represent the energy emitted by matter as a result of the changes in the electronic configurations of the atoms or molecules.

Electromagnetic waves transport energy just like other waves and they are characterized by their **frequency  $\nu$**  or **wavelength  $\lambda$** . These two properties in a medium are related by

$$\lambda = \frac{c}{\nu}$$

$$c = c_0 / n$$

$c$ , the speed of propagation of a wave in that medium

$c_0 = 2.9979 \times 10^8$  m/s, the *speed of light* in a vacuum

$n$ , the *index of refraction* of that medium

$n = 1$  for air and most gases,  $n = 1.5$  for glass, and  $n = 1.33$  for water

It has proven useful to view electromagnetic radiation as the propagation of a collection of discrete packets of energy called **photons** or **quanta**.

In this view, each photon of frequency  $\nu$  is considered to have an energy of

$$e = h\nu = \frac{hc}{\lambda}$$

The energy of a photon is inversely proportional to its wavelength.

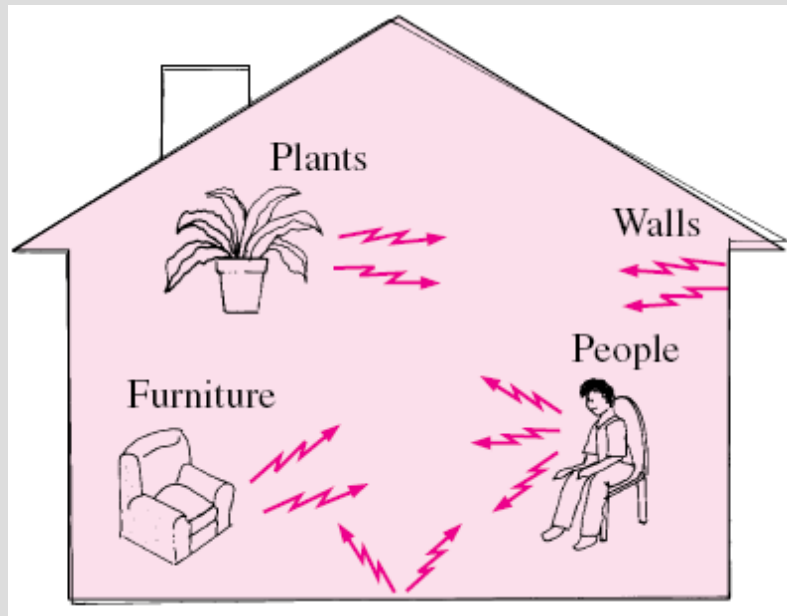
$h = 6.626069 \times 10^{-34}$  J · s is *Planck's constant*.

# THERMAL RADIATION

The type of electromagnetic radiation that is pertinent to heat transfer is the **thermal radiation** emitted as a result of energy transitions of molecules, atoms, and electrons of a substance.

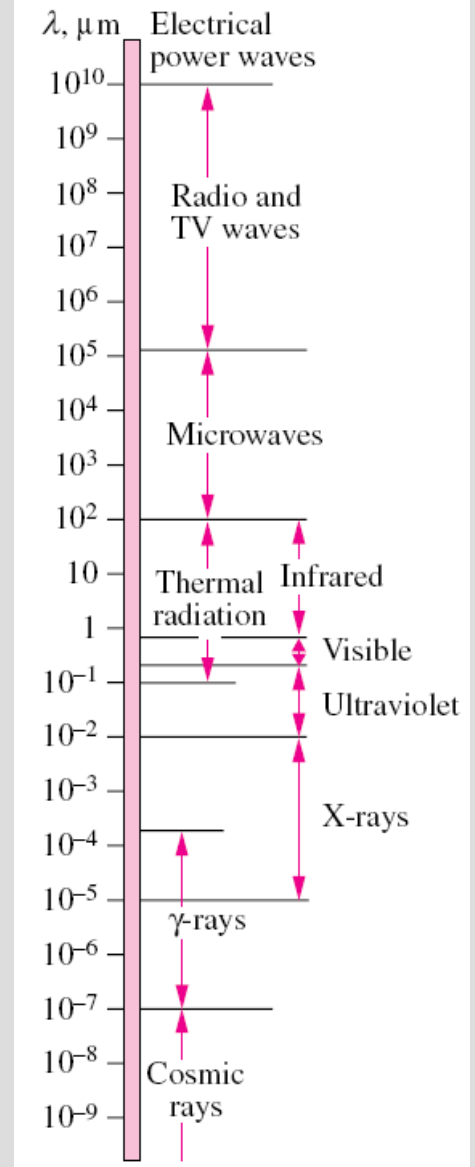
Temperature is a measure of the strength of these activities at the microscopic level, and the rate of thermal radiation emission increases with increasing temperature.

Thermal radiation is continuously emitted by all matter whose temperature is above absolute zero.



Everything around us constantly emits thermal radiation.

The electromagnetic wave spectrum.



**Light** is simply the *visible* portion of the electromagnetic spectrum that lies between 0.40 and 0.76  $\mu\text{m}$ .

**TABLE 12–1**

The wavelength ranges of different colors

Color	Wavelength band
Violet	0.40–0.44 $\mu\text{m}$
Blue	0.44–0.49 $\mu\text{m}$
Green	0.49–0.54 $\mu\text{m}$
Yellow	0.54–0.60 $\mu\text{m}$
Orange	0.60–0.67 $\mu\text{m}$
Red	0.63–0.76 $\mu\text{m}$

A body that emits some radiation in the visible range is called a **light source**.

The sun is our primary light source.

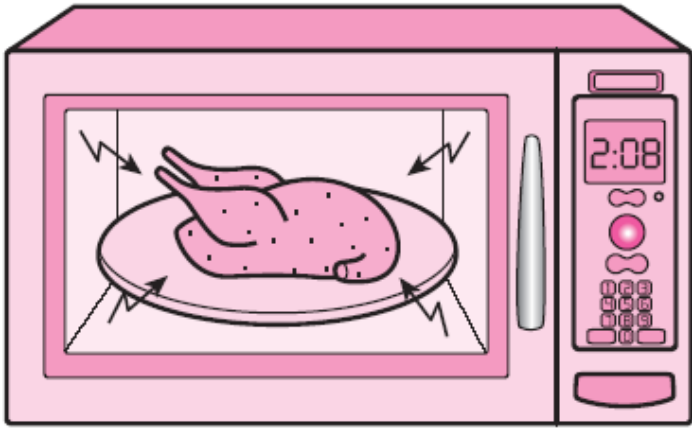
The electromagnetic radiation emitted by the sun is known as **solar radiation**, and nearly all of it falls into the wavelength band 0.3–3  $\mu\text{m}$ .

Almost *half* of solar radiation is **light** (i.e., it falls into the visible range), with the remaining being **ultraviolet** and **infrared**.

The radiation emitted by bodies at room temperature falls into the **infrared** region of the spectrum, which extends from 0.76 to 100  $\mu\text{m}$ .

The **ultraviolet** radiation includes the low-wavelength end of the thermal radiation spectrum and lies between the wavelengths 0.01 and 0.40  $\mu\text{m}$ . Ultraviolet rays are to be avoided since they can kill microorganisms and cause serious damage to humans and other living beings.

About 12 percent of solar radiation is in the ultraviolet range. The **ozone ( $\text{O}_3$ ) layer** in the atmosphere acts as a protective blanket and absorbs most of this ultraviolet radiation.



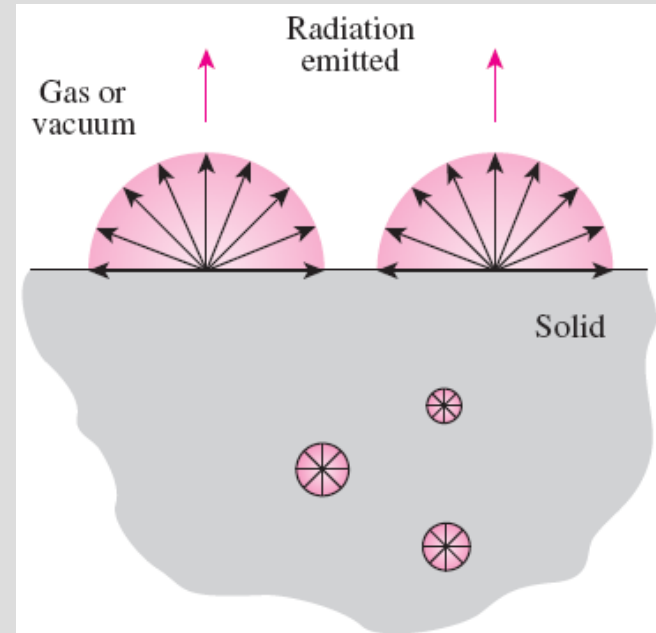
**FIGURE 12-5**

Food is heated or cooked in a microwave oven by absorbing the electromagnetic radiation energy generated by the magnetron of the oven.

The electrons, atoms, and molecules of all solids, liquids, and gases above absolute zero temperature are constantly in motion, and thus radiation is constantly emitted, as well as being absorbed or transmitted throughout the entire volume of matter.

That is, radiation is a **volumetric phenomenon**.

In heat transfer studies, we are interested in the energy emitted by bodies because of their temperature only. Therefore, we limit our consideration to **thermal radiation**.



**FIGURE 12-6**

Radiation in opaque solids is considered a surface phenomenon since the radiation emitted only by the molecules at the surface can escape the solid.

# BLACKBODY RADIATION

- Different bodies may emit different amounts of radiation per unit surface area.
- A **blackbody** emits the *maximum* amount of radiation by a surface at a given temperature.
- It is an *idealized body* to serve as a standard against which the radiative properties of real surfaces may be compared.
- A blackbody is a *perfect emitter and absorber of radiation*.
- A blackbody absorbs *all* incident radiation, regardless of wavelength and direction.

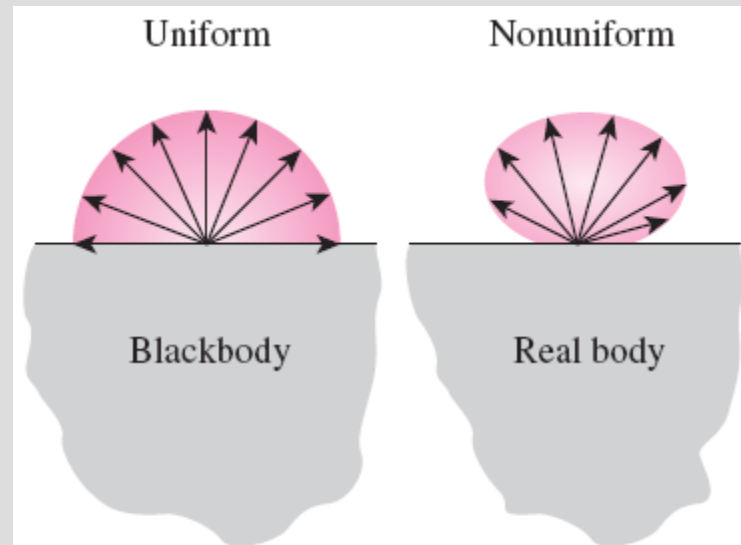
The radiation energy emitted by a blackbody:

$$E_b(T) = \sigma T^4 \quad (\text{W/m}^2)$$

*Blackbody emissive power*

$$\sigma = 5.670 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4$$

*Stefan–Boltzmann constant*



**FIGURE 12–7**

A blackbody is said to be a *diffuse* emitter since it emits radiation energy uniformly in all directions.



## Spectral blackbody emissive Power:

The amount of radiation energy emitted by a blackbody at a thermodynamic temperature  $T$  per unit time, per unit surface area, and per unit wavelength about the wavelength  $\lambda$ .

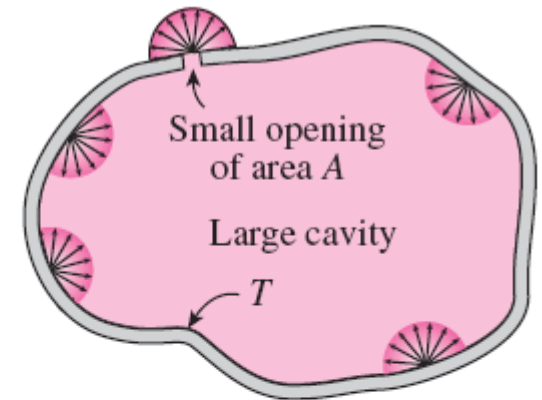
$$E_{b\lambda}(\lambda, T) = \frac{C_1}{\lambda^5 [\exp(C_2/\lambda T) - 1]} \quad (\text{W/m}^2 \cdot \mu\text{m})$$

Planck's  
law

$$C_1 = 2\pi hc_0^2 = 3.74177 \times 10^8 \text{ W} \cdot \mu\text{m}^4/\text{m}^2$$

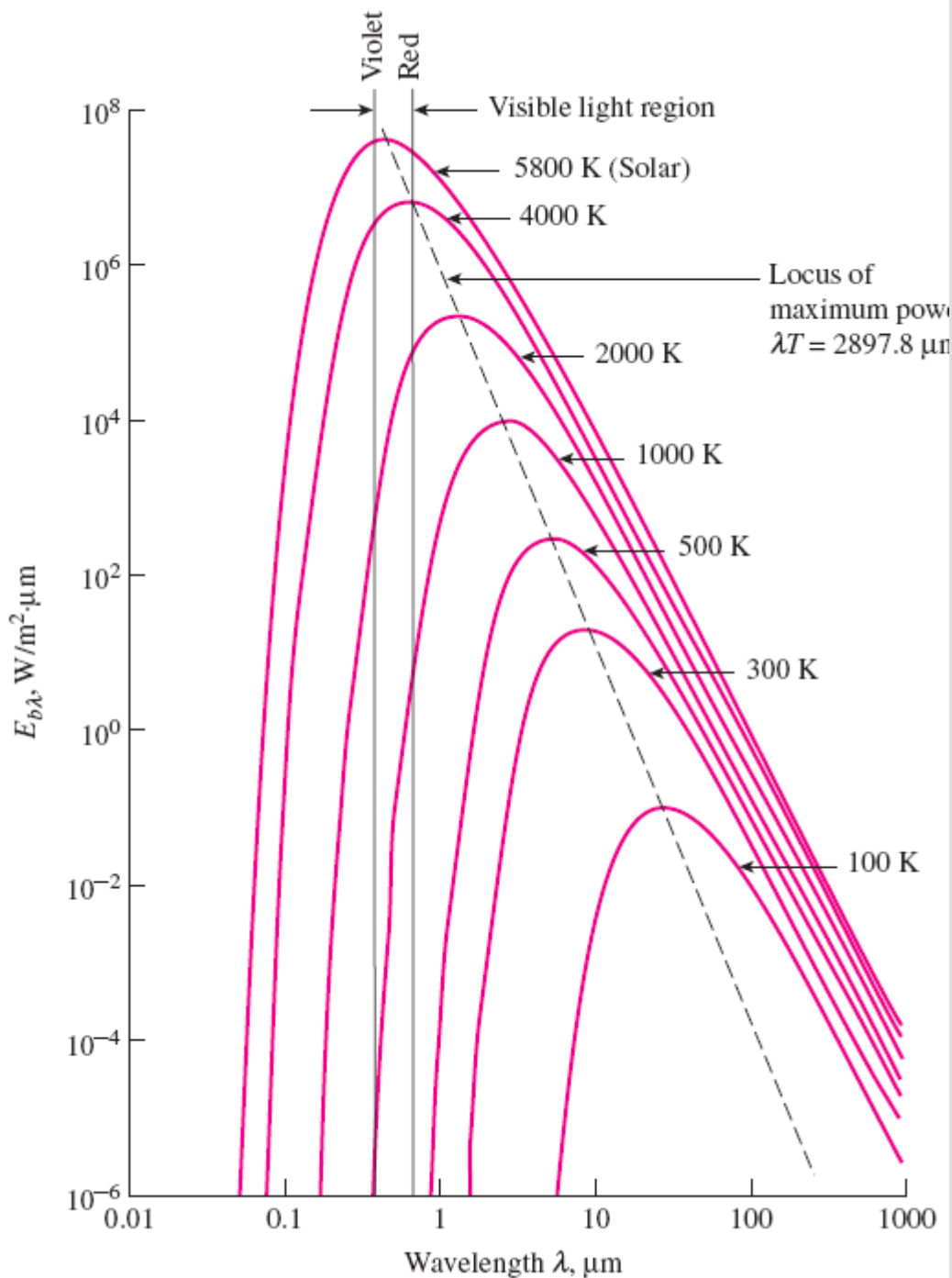
$$C_2 = hc_0/k = 1.43878 \times 10^4 \mu\text{m} \cdot \text{K}$$

$$k = 1.38065 \times 10^{-23} \text{ J/K} \quad \text{Boltzmann's constant}$$



**FIGURE 12-8**

A large isothermal cavity at temperature  $T$  with a small opening of area  $A$  closely resembles a blackbody of surface area  $A$  at the same temperature.



The wavelength at which the peak occurs for a specified temperature is given by **Wien's displacement law:**

$$(\lambda T)_{\text{max power}} = 2897.8 \mu\text{m} \cdot \text{K}$$

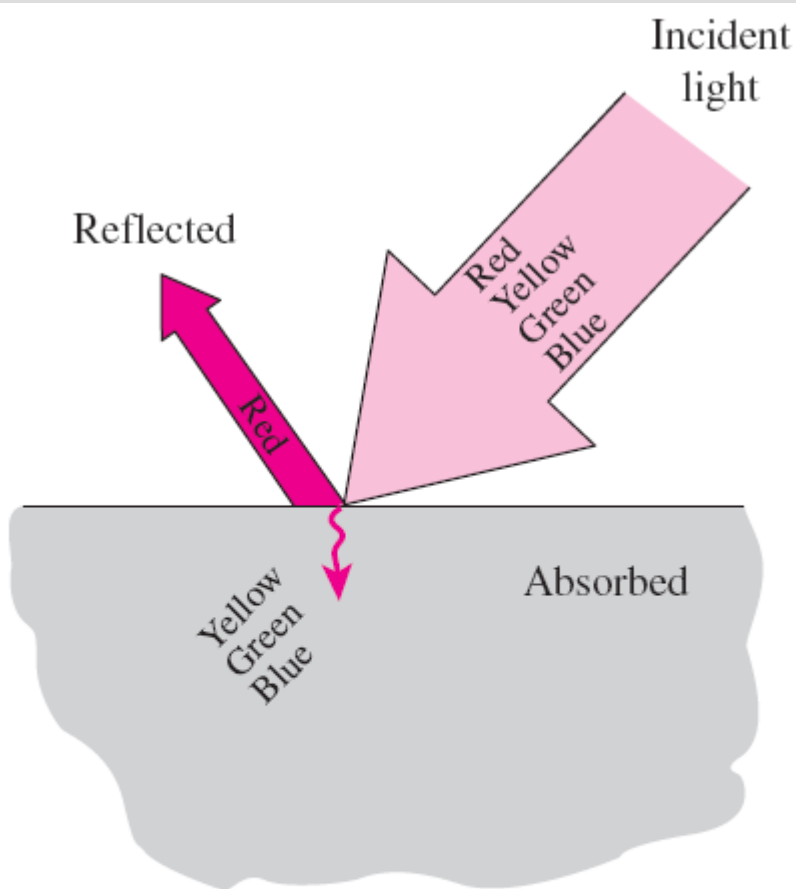
**FIGURE 12-9**

The variation of the blackbody emissive power with wavelength for several temperatures.

## Observations from the figure

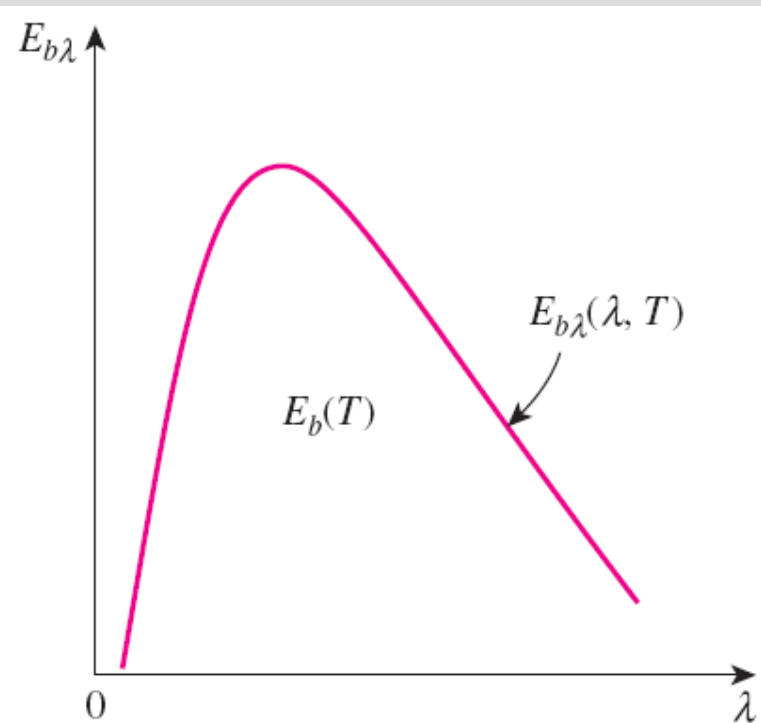
- The emitted radiation is a continuous function of *wavelength*. At any specified temperature, it increases with wavelength, reaches a peak, and then decreases with increasing wavelength.
- At any wavelength, the amount of emitted radiation *increases* with increasing temperature.
- As temperature increases, the curves shift to the left to the shorter wavelength region. Consequently, a larger fraction of the radiation is emitted at *shorter wavelengths* at higher temperatures.
- The radiation emitted by the *sun*, which is considered to be a blackbody at 5780 K (or roughly at 5800 K), reaches its peak in the visible region of the spectrum. Therefore, the sun is in tune with our eyes.
- On the other hand, surfaces at  $T < 800 \text{ K}$  emit almost entirely in the infrared region and thus are not visible to the eye unless they reflect light coming from other sources.

$$E_b(T) = \int_0^{\infty} E_{b\lambda}(\lambda, T) d\lambda = \sigma T^4 \quad (\text{W/m}^2)$$



**FIGURE 12-10**

A surface that reflects red while absorbing the remaining parts of the incident light appears red to the eye.

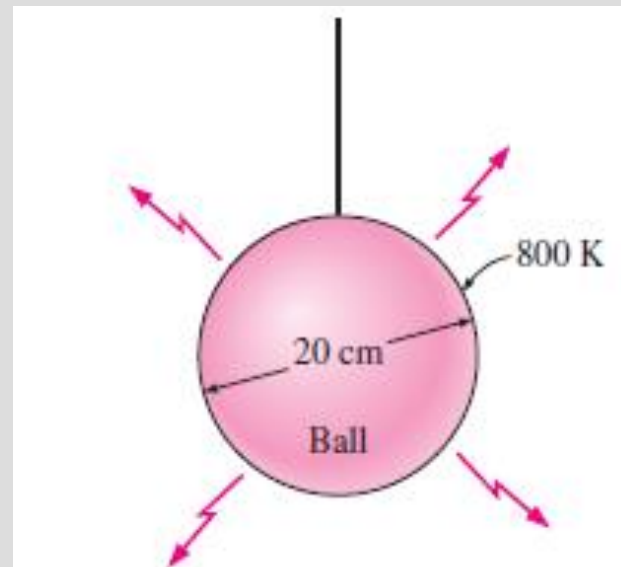


**FIGURE 12-11**

On an  $E_{b\lambda}-\lambda$  chart, the area under a curve for a given temperature represents the total radiation energy emitted by a blackbody at that temperature.

### **EXAMPLE 11–1**      **Radiation Emission from a Black Ball**

Consider a 20-cm-diameter spherical ball at 800 K suspended in air as shown in Figure 11–12. Assuming the ball closely approximates a blackbody, determine (a) the total blackbody emissive power, (b) the total amount of radiation emitted by the ball in 5 min, and (c) the spectral blackbody emissive power at a wavelength of  $3 \mu\text{m}$ .



**FIGURE 11–12**  
The spherical ball considered in Example 11–1.

**SOLUTION** An isothermal sphere is suspended in air. The total blackbody emissive power, the total radiation emitted in 5 minutes, and the spectral blackbody emissive power at 3 mm are to be determined.

**Assumptions** The ball behaves as a blackbody.

**Analysis** (a) The total blackbody emissive power is determined from the Stefan–Boltzmann law to be

$$E_b = \sigma T^4 = (5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)(800 \text{ K})^4 = 23.2 \times 10^3 \text{ W/m}^2 = 23.2 \text{ kW/m}^2$$

That is, the ball emits 23.2 kJ of energy in the form of electromagnetic radiation per second per m<sup>2</sup> of the surface area of the ball.

(b) The total amount of radiation energy emitted from the entire ball in 5 min is determined by multiplying the blackbody emissive power obtained above by the total surface area of the ball and the given time interval:

$$A_s = \pi D^2 = \pi(0.2 \text{ m})^2 = 0.1257 \text{ m}^2$$

$$\Delta t = (5 \text{ min})\left(\frac{60 \text{ s}}{1 \text{ min}}\right) = 300 \text{ s}$$

$$\begin{aligned} Q_{\text{rad}} &= E_b A_s \Delta t = (23.2 \text{ kW/m}^2)(0.1257 \text{ m}^2)(300 \text{ s})\left(\frac{1 \text{ kJ}}{1000 \text{ W} \cdot \text{s}}\right) \\ &= 876 \text{ kJ} \end{aligned}$$

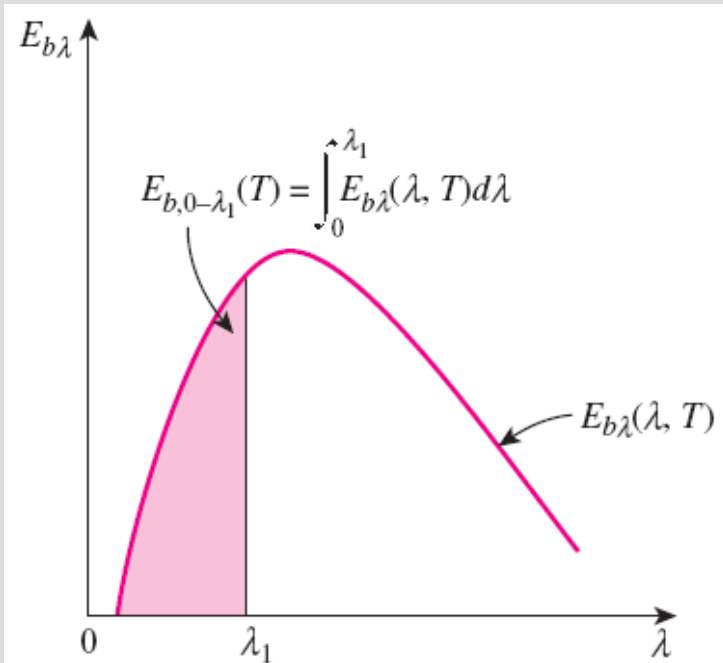
That is, the ball loses 876 kJ of its internal energy in the form of electromagnetic waves to the surroundings in 5 min, which is enough energy to raise the temperature of 1 kg of water by 50°C. Note that the surface temperature of the ball cannot remain constant at 800 K unless there is an equal amount of energy flow to the surface from the surroundings or from the interior regions of the ball through some mechanisms such as chemical or nuclear reactions.

(c) The spectral blackbody emissive power at a wavelength of 3 μm is determined from Planck's distribution law to be

$$\begin{aligned} E_{b\lambda} &= \frac{C_1}{\lambda^5 \left[ \exp\left(\frac{C_2}{\lambda T}\right) - 1 \right]} = \frac{3.743 \times 10^8 \text{ W} \cdot \mu\text{m}^4/\text{m}^2}{(3 \mu\text{m})^5 \left[ \exp\left(\frac{1.4387 \times 10^4 \mu\text{m} \cdot \text{K}}{(3 \mu\text{m})(800 \text{ K})}\right) - 1 \right]} \\ &= 3848 \text{ W/m}^2 \cdot \mu\text{m} \end{aligned}$$

The radiation energy emitted by a blackbody per unit area over a wavelength band from  $\lambda = 0$  to  $\lambda$  is

$$E_{b,0-\lambda}(T) = \int_0^{\lambda} E_{b\lambda}(\lambda, T) d\lambda \quad (\text{W/m}^2)$$



**FIGURE 12-13**

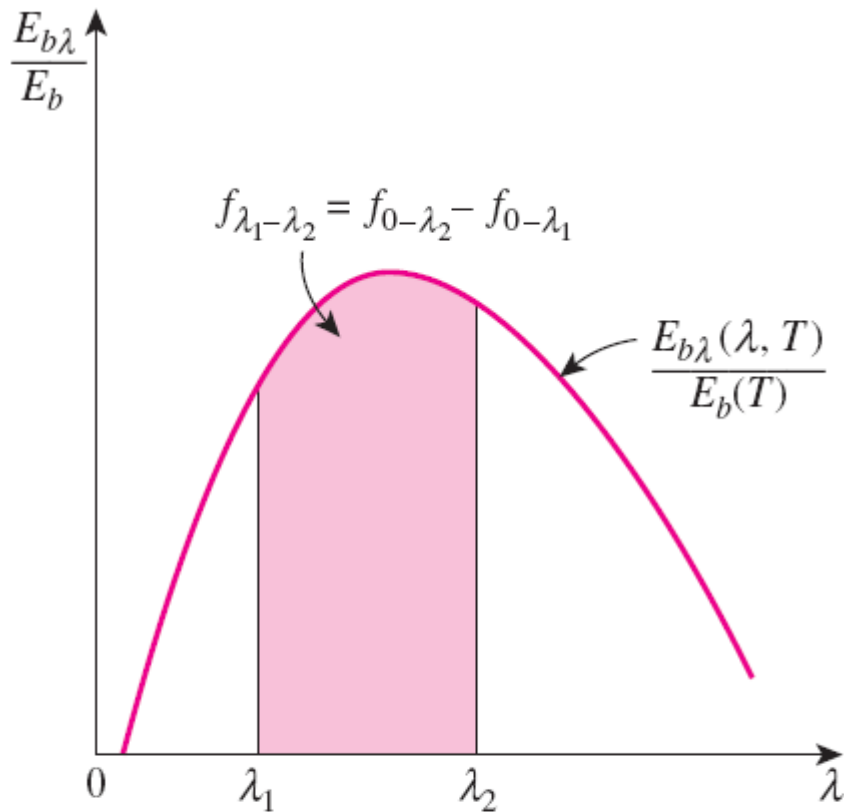
On an  $E_{b\lambda}$ - $\lambda$  chart, the area under the curve to the left of the  $\lambda = \lambda_1$  line represents the radiation energy emitted by a blackbody in the wavelength range  $0-\lambda_1$  for the given temperature.

**Blackbody radiation function  $f_\lambda$ :**  
 The fraction of radiation emitted from a blackbody at temperature  $T$  in the wavelength band from  $\lambda = 0$  to  $\lambda$ .

$$f_\lambda(T) = \frac{\int_0^{\lambda} E_{b\lambda}(\lambda, T) d\lambda}{\sigma T^4}$$

$$f_{\lambda_1-\lambda_2}(T) = f_{\lambda_2}(T) - f_{\lambda_1}(T)$$

$$f_{\lambda_1-\lambda_2}(T) = \frac{\int_0^{\lambda_2} E_{b\lambda}(\lambda, T)d\lambda - \int_0^{\lambda_1} E_{b\lambda}(\lambda, T)d\lambda}{\sigma T^4} = f_{\lambda_2}(T) - f_{\lambda_1}(T)$$



**FIGURE 12-14**

Graphical representation of the fraction of radiation emitted in the wavelength band from  $\lambda_1$  to  $\lambda_2$ .



**TABLE 12-2**

 Blackbody radiation functions  $f_\lambda$ 

$\lambda T, \mu\text{m}\cdot\text{K}$	$f_\lambda$	$\lambda T, \mu\text{m}\cdot\text{K}$	$f_\lambda$
200	0.000000	6200	0.754140
400	0.000000	6400	0.769234
600	0.000000	6600	0.783199
800	0.000016	6800	0.796129
1000	0.000321	7000	0.808109
1200	0.002134	7200	0.819217
1400	0.007790	7400	0.829527
1600	0.019718	7600	0.839102
1800	0.039341	7800	0.848005
2000	0.066728	8000	0.856288
2200	0.100888	8500	0.874608
2400	0.140256	9000	0.890029
2600	0.183120	9500	0.903085
2800	0.227897	10,000	0.914199
3000	0.273232	10,500	0.923710
3200	0.318102	11,000	0.931890
3400	0.361735	11,500	0.939959
3600	0.403607	12,000	0.945098
3800	0.443382	13,000	0.955139
4000	0.480877	14,000	0.962898
4200	0.516014	15,000	0.969981
4400	0.548796	16,000	0.973814
4600	0.579280	18,000	0.980860
4800	0.607559	20,000	0.985602
5000	0.633747	25,000	0.992215
5200	0.658970	30,000	0.995340
5400	0.680360	40,000	0.997967
5600	0.701046	50,000	0.998953
5800	0.720158	75,000	0.999713
6000	0.737818	100,000	0.999905

### **EXAMPLE 11-2**      **Emission of Radiation from a Lightbulb**

The temperature of the filament of an incandescent lightbulb is 2500 K. Assuming the filament to be a blackbody, determine the fraction of the radiant energy emitted by the filament that falls in the visible range. Also, determine the wavelength at which the emission of radiation from the filament peaks.

**SOLUTION** The temperature of the filament of an incandescent lightbulb is given. The fraction of visible radiation emitted by the filament and the wavelength at which the emission peaks are to be determined.

**Assumptions** The filament behaves as a blackbody.

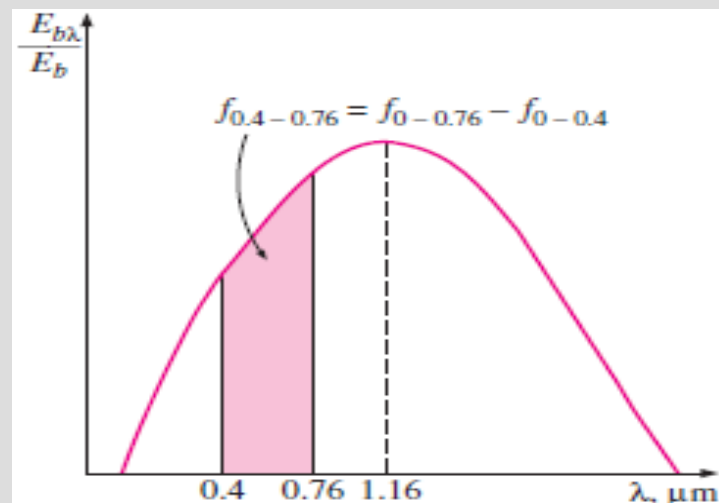
**Analysis** The visible range of the electromagnetic spectrum extends from  $\lambda_1 = 0.4 \mu\text{m}$  to  $\lambda_2 = 0.76 \mu\text{m}$ . Noting that  $T = 2500 \text{ K}$ , the blackbody radiation functions corresponding to  $\lambda_1 T$  and  $\lambda_2 T$  are determined from Table 11–2 to be

$$\lambda_1 T = (0.40 \mu\text{m})(2500 \text{ K}) = 1000 \mu\text{m} \cdot \text{K} \longrightarrow f_{\lambda_1} = 0.000321$$

$$\lambda_2 T = (0.76 \mu\text{m})(2500 \text{ K}) = 1900 \mu\text{m} \cdot \text{K} \longrightarrow f_{\lambda_2} = 0.053035$$

That is, 0.03 percent of the radiation is emitted at wavelengths less than  $0.4 \mu\text{m}$  and 5.3 percent at wavelengths less than  $0.76 \mu\text{m}$ . Then the fraction of radiation emitted between these two wavelengths is (Fig. 11–15)

$$f_{\lambda_1-\lambda_2} = f_{\lambda_2} - f_{\lambda_1} = 0.053035 - 0.000321 = \mathbf{0.0527135}$$



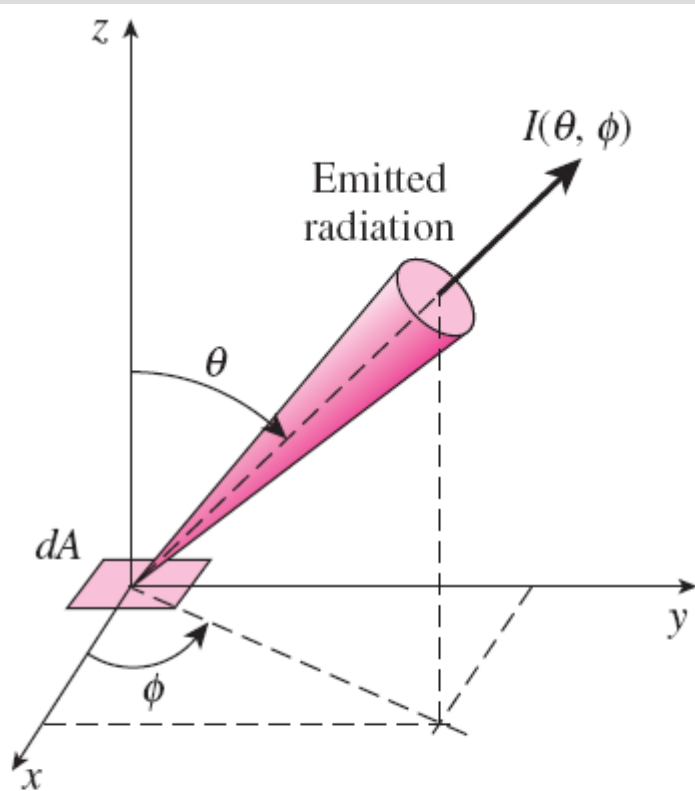
**FIGURE 11–15**

Therefore, only about 5 percent of the radiation emitted by the filament of the lightbulb falls in the visible range. The remaining 95 percent of the radiation appears in the infrared region in the form of radiant heat or “invisible light,” as it used to be called. This is certainly not a very efficient way of converting electrical energy to light and explains why fluorescent tubes are a wiser choice for lighting.

The wavelength at which the emission of radiation from the filament peaks is easily determined from Wien's displacement law to be

$$(\lambda T)_{\max \text{ power}} = 2897.8 \mu\text{m} \cdot \text{K} \quad \rightarrow \quad \lambda_{\max \text{ power}} = \frac{2897.8 \mu\text{m} \cdot \text{K}}{2500 \text{ K}} = 1.16 \mu\text{m}$$

# RADIATION INTENSITY



**FIGURE 12-16**

Radiation intensity is used to describe the variation of radiation energy with direction.

Radiation is emitted by all parts of a plane surface in all directions into the hemisphere above the surface, and the directional distribution of emitted (or incident) radiation is usually not uniform.

Therefore, we need a quantity that describes the magnitude of radiation emitted (or incident) in a specified direction in space.

This quantity is *radiation intensity*, denoted by  $I$ .

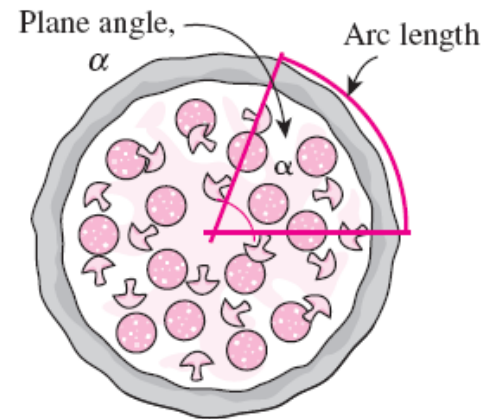
# Solid Angle

Now consider a watermelon, and let us attempt to quantify the size of a slice. Again we can do it by specifying the outer surface area of the slice (the green part), or by working with angles for generality. Connecting all points at the edges of the slice to the center in this case will form a three-dimensional body (like a cone whose tip is at the center), and thus the angle at the center in this case is properly called the **solid angle**. The solid angle is denoted by  $\omega$ , and its unit is the *steradian* (sr). In analogy to plane angle, we can say that *the area of a surface on a sphere of unit radius is equivalent in magnitude to the solid angle it subtends* (both are  $4\pi$  for a sphere of radius  $r = 1$ ).

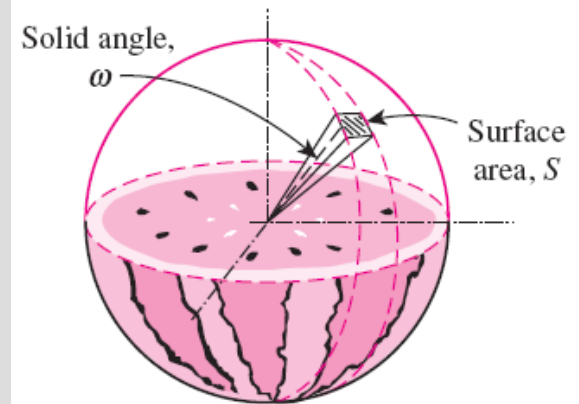
This can be shown easily by considering a differential surface area on a sphere  $dS = r^2 \sin \theta d\theta d\phi$ , as shown in Fig. 12-18, and integrating it from  $\theta = 0$  to  $\theta = \pi$ , and from  $\phi = 0$  to  $\phi = 2\pi$ . We get

$$S = \int_{\text{sphere}} dS = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} r^2 \sin \theta d\theta d\phi = 2\pi r^2 \int_{\theta=0}^{\pi} \sin \theta d\theta = 4\pi r^2 \quad (12-10)$$

which is the formula for the area of a sphere. For  $r = 1$  it reduces to  $S = 4\pi$ , and thus the solid angle associated with a sphere is  $\omega = 4\pi$  sr. For a hemisphere, which is more relevant to radiation emitted or received by a surface, it is  $\omega = 2\pi$  sr.



A slice of pizza of plane angle  $\alpha$



A slice of watermelon of solid angle  $\omega$

**FIGURE 12-17**

Describing the size of a slice of pizza by a plane angle, and the size of a watermelon slice by a solid angle.

The differential solid angle  $d\omega$  subtended by a differential area  $dS$  on a sphere of radius  $r$  can be expressed as

$$d\omega = \frac{dS}{r^2} = \sin \theta \, d\theta \, d\phi \quad (12-11)$$

Note that the area  $dS$  is normal to the direction of viewing since  $dS$  is viewed from the center of the sphere. In general, the differential solid angle  $d\omega$  subtended by a differential surface area  $dA$  when viewed from a point at a distance  $r$  from  $dA$  is expressed as

$$d\omega = \frac{dA_n}{r^2} = \frac{dA \cos \alpha}{r^2} \quad (12-12)$$

where  $\alpha$  is the angle between the normal of the surface and the direction of viewing, and thus  $dA_n = dA \cos \alpha$  is the normal (or projected) area to the direction of viewing.

Small surfaces viewed from relatively large distances can approximately be treated as differential areas in solid angle calculations. For example, the solid angle subtended by a  $5 \text{ cm}^2$  plane surface when viewed from a point at a distance of 80 cm along the normal of the surface is

$$\omega \cong \frac{A_n}{r^2} = \frac{5 \text{ cm}^2}{(80 \text{ cm})^2} = 7.81 \times 10^{-4} \text{ sr}$$

If the surface is tilted so that the normal of the surface makes an angle of  $\alpha = 60^\circ$  with the line connecting the point of viewing to the center of the surface, the projected area would be  $dA_n = dA \cos \alpha = (5 \text{ cm}^2)\cos 60^\circ = 2.5 \text{ cm}^2$ , and the solid angle in this case would be half of the value just determined.

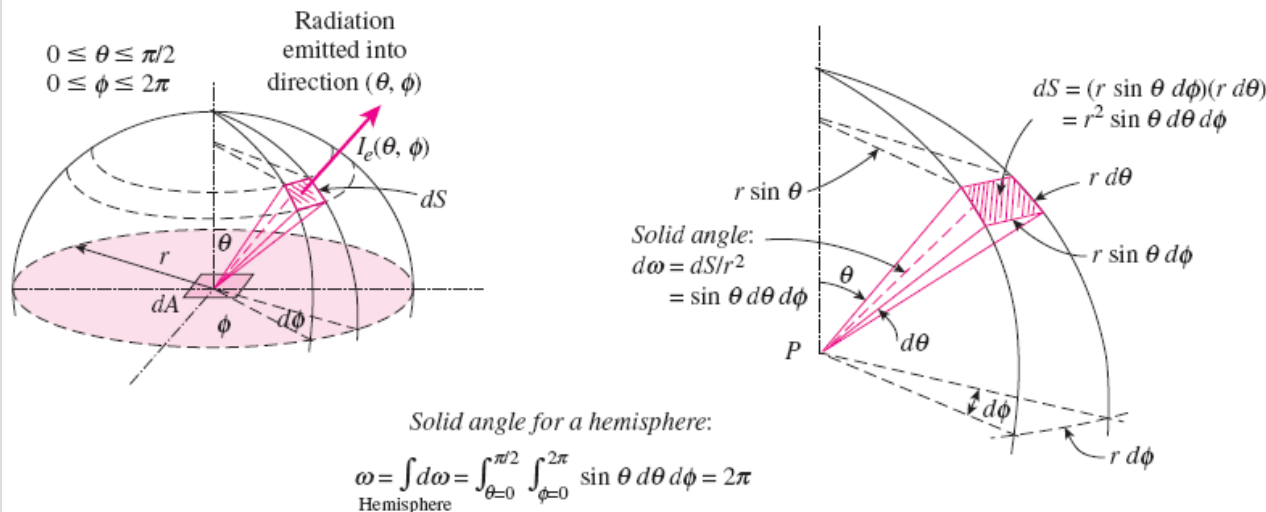
# Intensity of Emitted Radiation

The **radiation intensity** for emitted radiation  $I_e(\theta, \phi)$  is defined as *the rate at which radiation energy  $d\dot{Q}_e$  is emitted in the  $(\theta, \phi)$  direction per unit area normal to this direction and per unit solid angle about this direction*. That is,

$$I_e(\theta, \phi) = \frac{d\dot{Q}_e}{dA \cos \theta \cdot d\omega} = \frac{d\dot{Q}_e}{dA \cos \theta \sin \theta d\theta d\phi} \quad (\text{W/m}^2 \cdot \text{sr}) \quad (12-13)$$

The *radiation flux* for emitted radiation is the **emissive power  $E$**  (the rate at which radiation energy is emitted per unit area of the emitting surface), which can be expressed in differential form as

$$dE = \frac{d\dot{Q}_e}{dA} = I_e(\theta, \phi) \cos \theta \sin \theta d\theta d\phi \quad (12-14)$$



**FIGURE 12-18** The emission of radiation from a differential surface element into the surrounding hemispherical space through a differential solid angle.



Noting that the hemisphere above the surface intercepts all the radiation rays emitted by the surface, the emissive power from the surface into the hemisphere surrounding it can be determined by integration as

$$E = \int_{\text{hemisphere}} dE = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi/2} I_e(\theta, \phi) \cos \theta \sin \theta d\theta d\phi \quad (\text{W/m}^2) \quad (12-15)$$

The intensity of radiation emitted by a surface, in general, varies with direction (especially with the zenith angle  $\theta$ ). But many surfaces in practice can be approximated as being diffuse. For a *diffusely emitting* surface, the intensity of the emitted radiation is independent of direction and thus  $I_e = \text{constant}$ .

Noting that  $\int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi/2} \cos \theta \sin \theta d\theta d\phi = \pi$ , the emissive power relation in

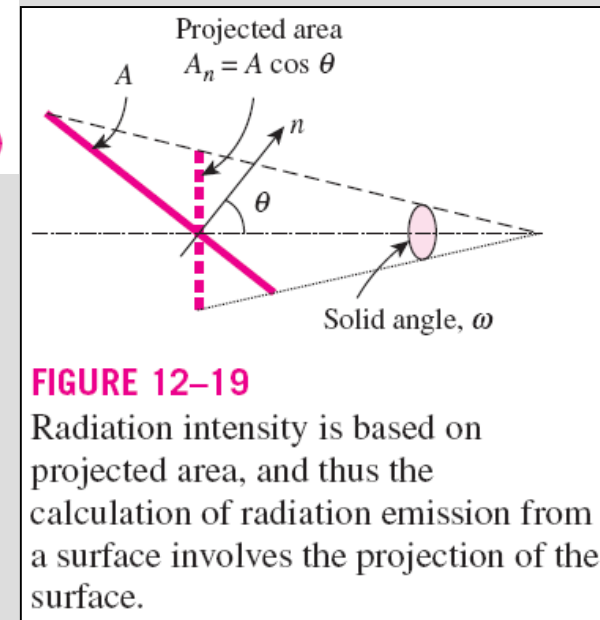
Eq. 12-15 reduces in this case to

$$\text{Diffusely emitting surface:} \quad E = \pi I_e \quad (\text{W/m}^2) \quad (12-16)$$

$$\text{Blackbody:} \quad E_b = \pi I_b$$

where  $E_b = \sigma T^4$  is the blackbody emissive power.

$$\text{Blackbody:} \quad I_b(T) = \frac{E_b(T)}{\pi} = \frac{\sigma T^4}{\pi} \quad (\text{W/m}^2 \cdot \text{sr})$$



**FIGURE 12-19** Radiation intensity is based on projected area, and thus the calculation of radiation emission from a surface involves the projection of the surface.

# Incident Radiation

All surfaces emit radiation, but they also receive radiation emitted or reflected by other surfaces. The intensity of incident radiation  $I_i(\theta, \phi)$  is defined as *the rate at which radiation energy  $dG$  is incident from the  $(\theta, \phi)$  direction per unit area of the receiving surface normal to this direction and per unit solid angle about this direction* (Fig. 12–20). Here  $\theta$  is the angle between the direction of incident radiation and the normal of the surface.

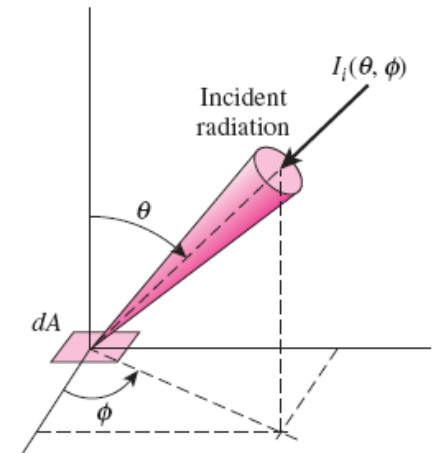
The radiation flux incident on a surface from *all directions* is called **irradiation**  $G$ , and is expressed as

$$G = \int_{\text{hemisphere}} dG = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi/2} I_i(\theta, \phi) \cos \theta \sin \theta \, d\theta \, d\phi \quad (\text{W/m}^2) \quad (12-19)$$

Therefore irradiation represents the rate at which radiation energy is incident on a surface per unit area of the surface. When the incident radiation is diffuse and thus  $I_i = \text{constant}$ , Eq. 12–19 reduces to

$$\text{Diffusely incident radiation:} \quad G = \pi I_i \quad (\text{W/m}^2) \quad (12-20)$$

Again note that irradiation is based on the *actual* surface area (and thus the factor  $\cos \theta$ ), whereas the intensity of incident radiation is based on the *projected* area.



**FIGURE 12-20**

Radiation incident on a surface in the direction  $(\theta, \phi)$ .

# Radiosity

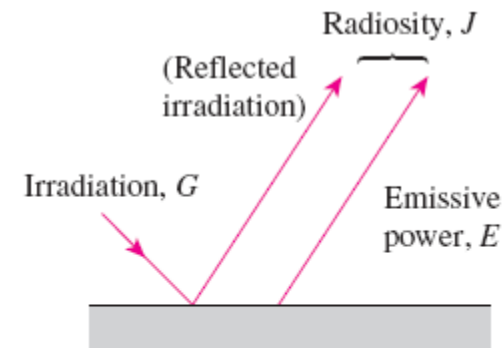
Surfaces emit radiation as well as reflecting it, and thus the radiation leaving a surface consists of emitted and reflected components, as shown in Fig. 12–21. The calculation of radiation heat transfer between surfaces involves the *total* radiation energy streaming away from a surface, with no regard for its origin. Thus, we need to define a quantity that represents *the rate at which radiation energy leaves a unit area of a surface in all directions*. This quantity is called the **radiosity**  $J$ , and is expressed as

$$J = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi/2} I_{e+r}(\theta, \phi) \cos \theta \sin \theta \, d\theta \, d\phi \quad (\text{W/m}^2) \quad (12-21)$$

where  $I_{e+r}$  is the sum of the emitted and reflected intensities. For a surface that is both a diffuse emitter and a diffuse reflector,  $I_{e+r} = \text{constant}$ , and the radiosity relation reduces to

*Diffuse emitter and reflector:*  $J = \pi I_{e+r} \quad (\text{W/m}^2) \quad (12-22)$

For a blackbody, radiosity  $J$  is equivalent to the emissive power  $E_b$  since a blackbody absorbs the entire radiation incident on it and there is no reflected component in radiosity.



**FIGURE 12–21**

The three kinds of radiation flux (in  $\text{W/m}^2$ ): emissive power, irradiation, and radiosity.

## Spectral Quantities

But sometimes it is necessary to consider the variation of radiation with wavelength as well as direction, and to express quantities at a certain wavelength  $\lambda$  or per unit wavelength interval about  $\lambda$ . Such quantities are referred to as *spectral* quantities to draw attention to wavelength dependence. The modifier “spectral” is used to indicate “at a given wavelength.”

The *spectral radiation intensity*  $I_\lambda(\lambda, \theta, \phi)$ , for example, is simply the total radiation intensity  $I(\theta, \phi)$  per unit wavelength interval about  $\lambda$ . The **spectral intensity** for emitted radiation  $I_{\lambda,e}(\lambda, \theta, \phi)$  can be defined as *the rate at which radiation energy  $d\dot{Q}_e$  is emitted at the wavelength  $\lambda$  in the  $(\theta, \phi)$  direction per unit area normal to this direction, per unit solid angle about this direction*, and it can be expressed as

$$I_{\lambda,e}(\lambda, \theta, \phi) = \frac{d\dot{Q}_e}{dA \cos \theta \cdot d\omega \cdot d\lambda} \quad (\text{W/m}^2 \cdot \text{sr} \cdot \mu\text{m}) \quad (12-23)$$

Then the *spectral emissive power* becomes

$$E_\lambda = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi/2} I_{\lambda,e}(\lambda, \theta, \phi) \cos \theta \sin \theta \, d\theta \, d\phi \quad (\text{W/m}^2) \quad (12-24)$$

Similar relations can be obtained for spectral irradiation  $G_\lambda$ , and spectral radiosity  $J_\lambda$  by replacing  $I_{\lambda,e}$  in this equation by  $I_{\lambda,i}$  and  $I_{\lambda,e+r}$  respectively.

When the variation of spectral radiation intensity  $I_\lambda$  with wavelength  $\lambda$  is known, the total radiation intensity  $I$  for emitted, incident, and emitted + reflected radiation can be determined by integration over the entire wavelength spectrum as (Fig. 12–22)

$$I_e = \int_0^\infty I_{\lambda,e} d\lambda, \quad I_i = \int_0^\infty I_{\lambda,i} d\lambda, \quad \text{and} \quad I_{e+r} = \int_0^\infty I_{\lambda,e+r} d\lambda \quad (12-25)$$

These intensities can then be used in Eqs. 12–15, 12–19, and 12–21 to determine the emissive power  $E$ , irradiation  $G$ , and radiosity  $J$ , respectively.

Similarly, when the variations of spectral radiation fluxes  $E_\lambda$ ,  $G_\lambda$ , and  $J_\lambda$  with wavelength  $\lambda$  are known, the total radiation fluxes can be determined by integration over the entire wavelength spectrum as

$$E = \int_0^\infty E_\lambda d\lambda, \quad G = \int_0^\infty G_\lambda d\lambda, \quad \text{and} \quad J = \int_0^\infty J_\lambda d\lambda \quad (12-26)$$

When the surfaces and the incident radiation are *diffuse*, the spectral radiation fluxes are related to spectral intensities as

$$E_\lambda = \pi I_{\lambda,e}, \quad G_\lambda = \pi I_{\lambda,i}, \quad \text{and} \quad J_\lambda = \pi I_{\lambda,e+r} \quad (12-27)$$

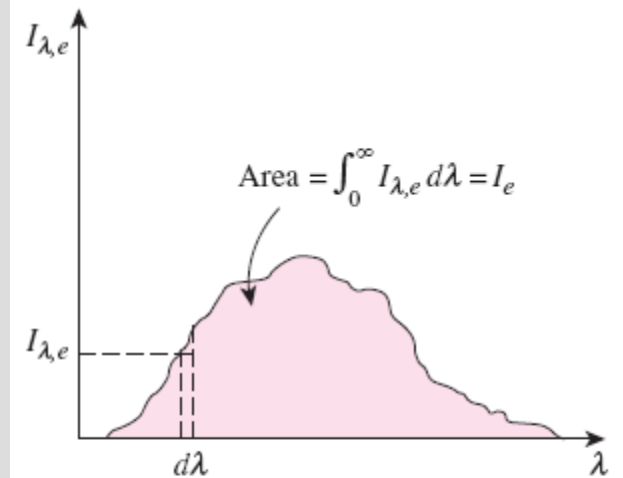
Note that the relations for spectral and total radiation quantities are of the same form.

The spectral intensity of radiation emitted by a blackbody at a thermodynamic temperature  $T$  at a wavelength  $\lambda$  has been determined by Max Planck, and is expressed as

$$I_{b\lambda}(\lambda, T) = \frac{2hc_0^2}{\lambda^5 [\exp(hc_0/\lambda kT) - 1]} \quad (\text{W/m}^2 \cdot \text{sr} \cdot \mu\text{m}) \quad (12-28)$$

where  $h = 6.626069 \times 10^{-34}$  J·s is the Planck constant,  $k = 1.38065 \times 10^{-23}$  J/K is the Boltzmann constant, and  $c_0 = 2.9979 \times 10^8$  m/s is the speed of light in a vacuum. Then the spectral blackbody emissive power is, from Eq. 12–27,

$$E_{b\lambda}(\lambda, T) = \pi I_{b\lambda}(\lambda, T) \quad (12-29)$$

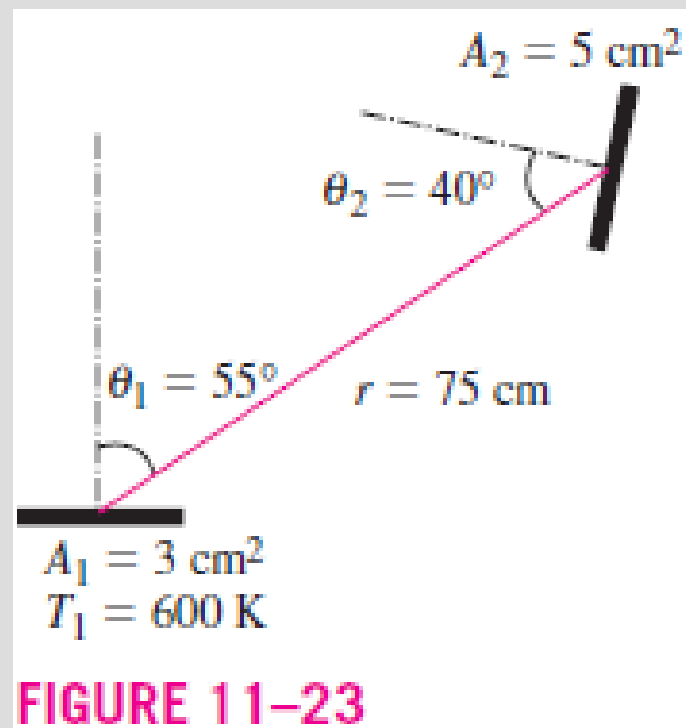


**FIGURE 12–22**

Integration of a “spectral” quantity for all wavelengths gives the “total” quantity.

### EXAMPLE 11-3 Radiation Incident on a Small Surface

A small surface of area  $A_1 = 3 \text{ cm}^2$  emits radiation as a blackbody at  $T_1 = 600 \text{ K}$ . Part of the radiation emitted by  $A_1$  strikes another small surface of area  $A_2 = 5 \text{ cm}^2$  oriented as shown in Figure 11-23. Determine the solid angle subtended by  $A_2$  when viewed from  $A_1$ , and the rate at which radiation emitted by  $A_1$  that strikes  $A_2$ .



**SOLUTION** A surface is subjected to radiation emitted by another surface. The solid angle subtended and the rate at which emitted radiation is received are to be determined.

**Assumptions** 1 Surface  $A_1$  emits diffusely as a blackbody. 2 Both  $A_1$  and  $A_2$  can be approximated as differential surfaces since both are very small compared to the square of the distance between them.

**Analysis** Approximating both  $A_1$  and  $A_2$  as differential surfaces, the solid angle subtended by  $A_2$  when viewed from  $A_1$  can be determined from Eq. 11-12 to be

$$\omega_{2-1} \cong \frac{A_{n,2}}{r^2} = \frac{A_2 \cos \theta_2}{r^2} = \frac{(5 \text{ cm}^2) \cos 40^\circ}{(75 \text{ cm})^2} = 6.81 \times 10^{-4} \text{ sr}$$

since the normal of  $A_2$  makes  $40^\circ$  with the direction of viewing. Note that solid angle subtended by  $A_2$  would be maximum if  $A_2$  were positioned normal to the direction of viewing. Also, the point of viewing on  $A_1$  is taken to be a point in the middle, but it can be any point since  $A_1$  is assumed to be very small.

The radiation emitted by  $A_1$  that strikes  $A_2$  is equivalent to the radiation emitted by  $A_1$  through the solid angle  $\omega_{2-1}$ . The intensity of the radiation emitted by  $A_1$  is

$$I_1 = \frac{E_b(T_1)}{\pi} = \frac{\sigma T_1^4}{\pi} = \frac{(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)(600 \text{ K})^4}{\pi} = 2339 \text{ W/m}^2 \cdot \text{sr}$$

This value of intensity is the same in all directions since a blackbody is a diffuse emitter. Intensity represents the rate of radiation emission per unit area normal to the direction of emission per unit solid angle. Therefore, the rate of radiation energy emitted by  $A_1$  in the direction of  $\theta_1$  through the solid angle  $\omega_{2-1}$  is determined by multiplying  $I_1$  by the area of  $A_1$  normal to  $\theta_1$  and the solid angle  $\omega_{2-1}$ . That is,

$$\begin{aligned} \dot{Q}_{1-2} &= I_1(A_1 \cos \theta_1)\omega_{2-1} \\ &= (2339 \text{ W/m}^2 \cdot \text{sr})(3 \times 10^{-4} \cos 55^\circ \text{ m}^2)(6.81 \times 10^{-4} \text{ sr}) \\ &= 2.74 \times 10^{-4} \text{ W} \end{aligned}$$

Therefore, the radiation emitted from surface  $A_1$  will strike surface  $A_2$  at a rate of  $2.74 \times 10^{-4} \text{ W}$ .



# RADIATIVE PROPERTIES

Most materials encountered in practice, such as metals, wood, and bricks, are *opaque* to thermal radiation, and radiation is considered to be a *surface phenomenon* for such materials.

Radiation through *semitransparent* materials such as glass and water cannot be considered to be a surface phenomenon since the entire volume of the material interacts with radiation.

A blackbody can serve as a convenient *reference* in describing the emission and absorption characteristics of real surfaces.

## Emissivity

- **Emissivity:** *The ratio of the radiation emitted by the surface at a given temperature to the radiation emitted by a blackbody at the same temperature.*  $0 \leq \varepsilon \leq 1$ .
- Emissivity is a measure of how closely a surface approximates a blackbody ( $\varepsilon = 1$ ).
- The emissivity of a real surface varies with the *temperature* of the surface as well as the *wavelength* and the *direction* of the emitted radiation.
- The emissivity of a surface at a specified wavelength is called *spectral emissivity*  $\varepsilon_\lambda$ . The emissivity in a specified direction is called *directional emissivity*  $\varepsilon_\theta$  where  $\theta$  is the angle between the direction of radiation and the normal of the surface.



$$\varepsilon_{\lambda, \theta}(\lambda, \theta, \phi, T) = \frac{I_{\lambda, e}(\lambda, \theta, \phi, T)}{I_{b\lambda}(\lambda, T)}$$

**spectral  
directional  
emissivity**

$$\varepsilon_{\theta}(\theta, \phi, T) = \frac{I_e(\theta, \phi, T)}{I_b(T)}$$

**total  
directional  
emissivity**

$$\varepsilon_{\lambda}(\lambda, T) = \frac{E_{\lambda}(\lambda, T)}{E_{b\lambda}(\lambda, T)}$$

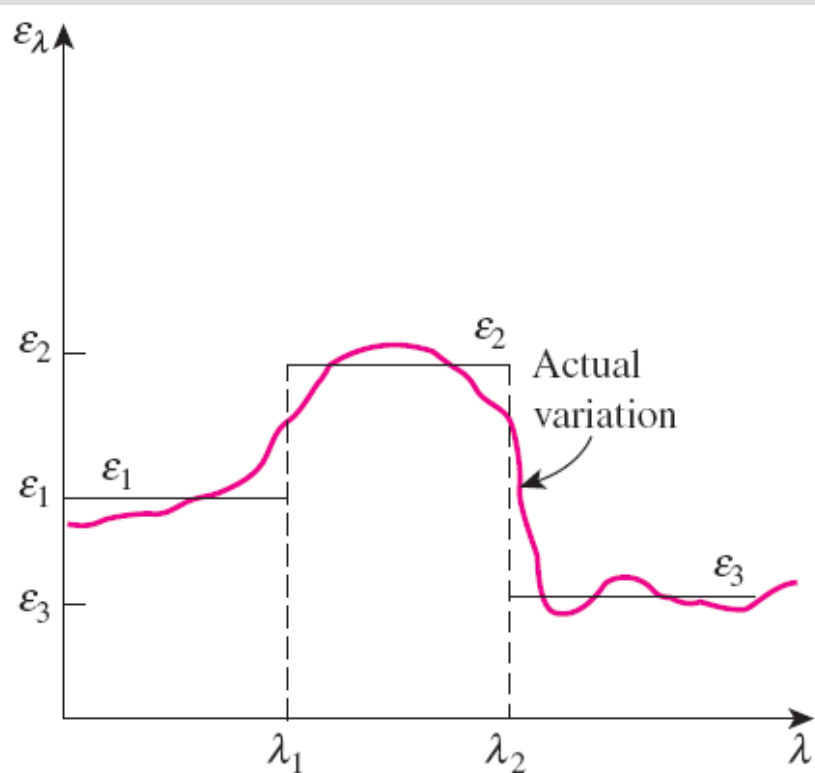
**spectral  
hemispherical  
emissivity**

$$\varepsilon(T) = \frac{E(T)}{E_b(T)}$$

**total  
hemispherical  
emissivity**

$$\varepsilon(T) = \frac{E(T)}{E_b(T)} = \frac{\int_0^{\infty} \varepsilon_{\lambda}(\lambda, T) E_{b\lambda}(\lambda, T) d\lambda}{\sigma T^4}$$

The ratio of the total radiation energy emitted by the surface to the radiation emitted by a blackbody of the same surface area at the same temperature



**FIGURE 12-24**

Approximating the actual variation of emissivity with wavelength by a step function.

$$\varepsilon_{\lambda} = \begin{cases} \varepsilon_1 = \text{constant}, & 0 \leq \lambda < \lambda_1 \\ \varepsilon_2 = \text{constant}, & \lambda_1 \leq \lambda < \lambda_2 \\ \varepsilon_3 = \text{constant}, & \lambda_2 \leq \lambda < \infty \end{cases}$$

$$\begin{aligned} \varepsilon(T) &= \frac{\varepsilon_1 \int_0^{\lambda_1} E_{b\lambda} d\lambda}{E_b} + \frac{\varepsilon_2 \int_{\lambda_1}^{\lambda_2} E_{b\lambda} d\lambda}{E_b} + \frac{\varepsilon_3 \int_{\lambda_2}^{\infty} E_{b\lambda} d\lambda}{E_b} \\ &= \varepsilon_1 f_{0-\lambda_1}(T) + \varepsilon_2 f_{\lambda_1-\lambda_2}(T) + \varepsilon_3 f_{\lambda_2-\infty}(T) \end{aligned}$$

A surface is said to be *diffuse* if its properties are *independent of direction*, and *gray* if its properties are *independent of wavelength*. The *gray* and *diffuse* approximations are often utilized in radiation calculations.

*Real surface:*

$$\epsilon_{\theta} \neq \text{constant}$$

$$\epsilon_{\lambda} \neq \text{constant}$$

*Diffuse surface:*

$$\epsilon_{\theta} = \text{constant}$$

*Gray surface:*

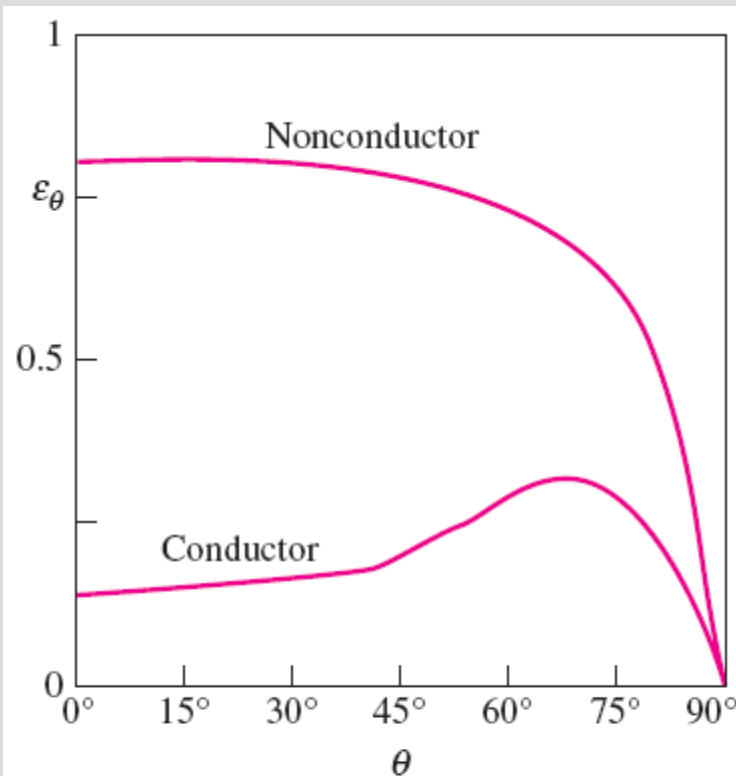
$$\epsilon_{\lambda} = \text{constant}$$

*Diffuse, gray surface:*

$$\epsilon = \epsilon_{\lambda} = \epsilon_{\theta} = \text{constant}$$

**FIGURE 12-25**

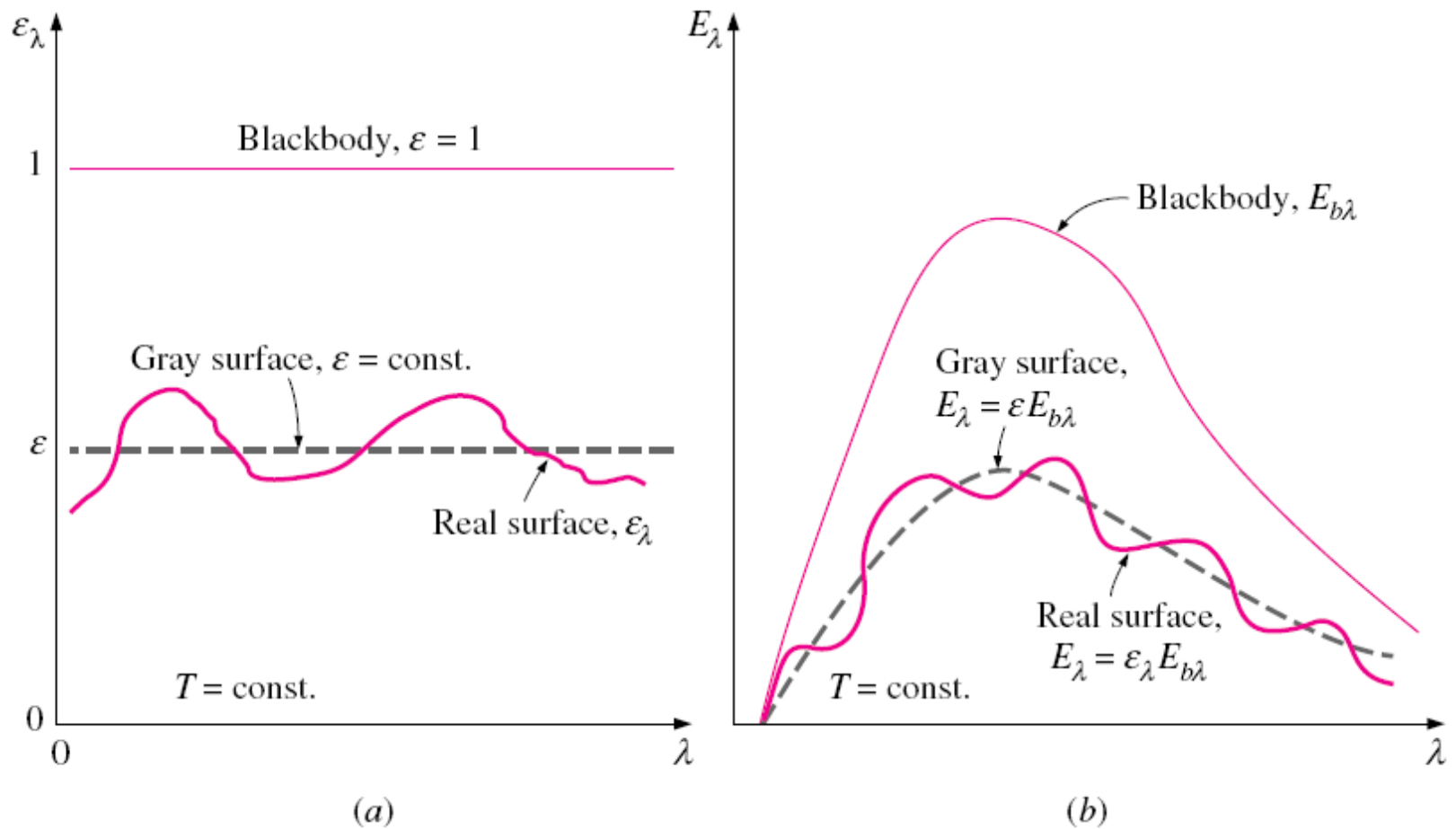
The effect of diffuse and gray approximations on the emissivity of a surface.



**FIGURE 12-26**

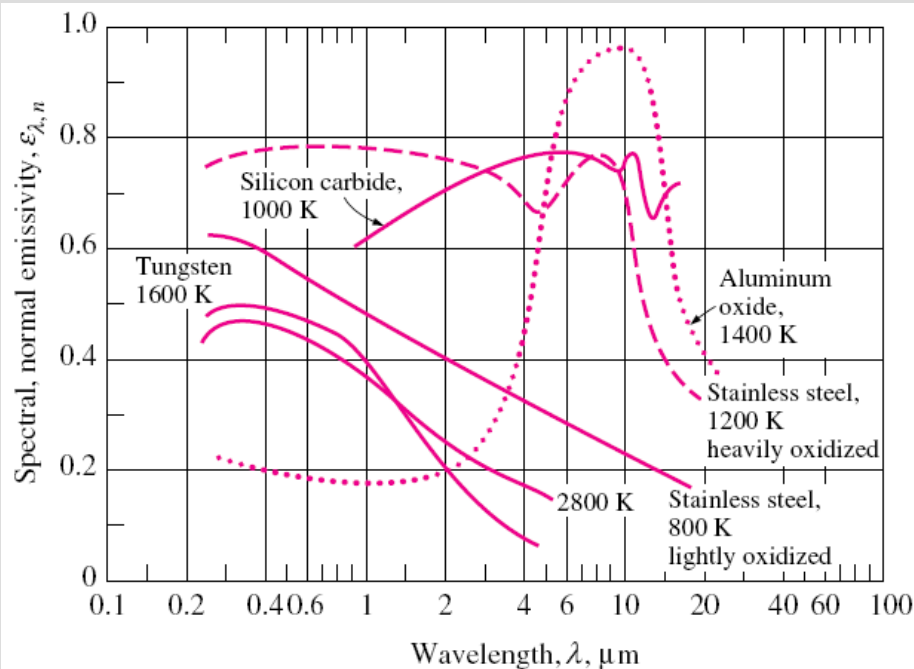
Typical variations of emissivity with direction for electrical conductors and nonconductors.

$\theta$  is the angle measured from the normal of the surface

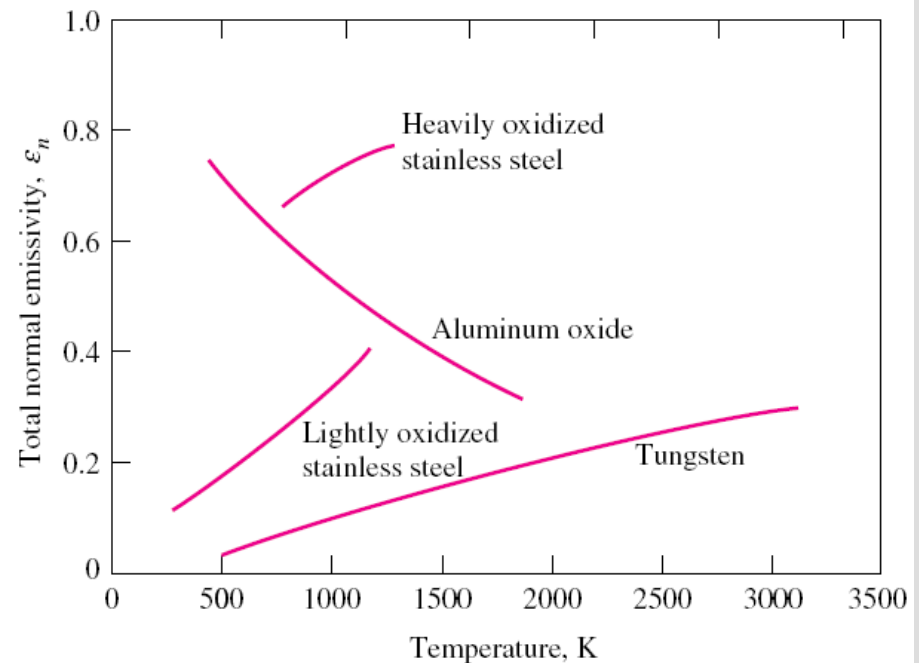


**FIGURE 12-27**

Comparison of the emissivity (a) and emissive power (b) of a real surface with those of a gray surface and a blackbody at the same temperature.



(a)

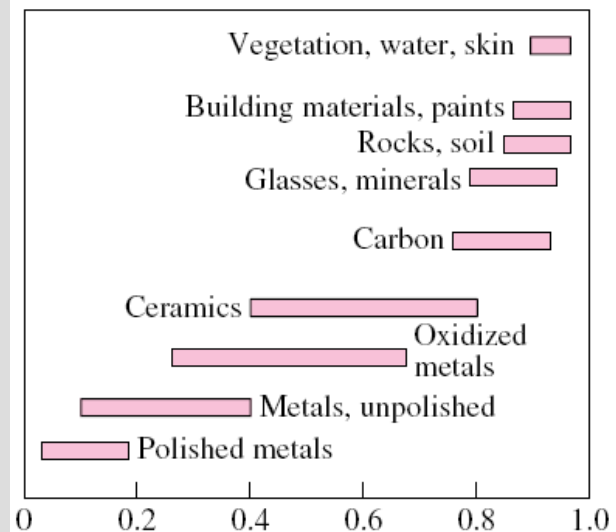


(b)

The variation of normal emissivity with (a) wavelength and (b) temperature for various materials.

In radiation analysis, it is common practice to assume the surfaces to be diffuse emitters with an emissivity equal to the value in the normal ( $\theta = 0$ ) direction.

Typical ranges of emissivity for various materials.

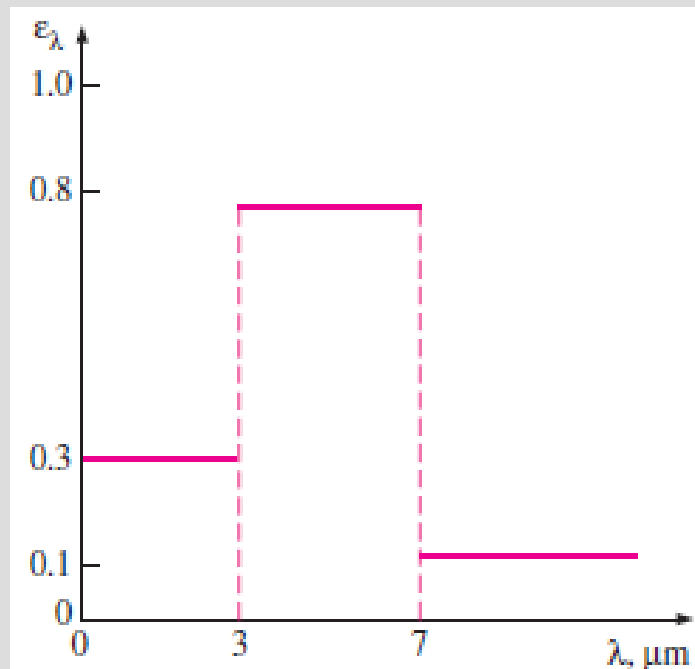


### EXAMPLE 11-4 Emissivity of a Surface and Emissive Power

The spectral emissivity function of an opaque surface at 800 K is approximated as (Fig. 11-30)

$$\varepsilon_\lambda = \begin{cases} \varepsilon_1 = 0.3, & 0 \leq \lambda < 3 \mu\text{m} \\ \varepsilon_2 = 0.8, & 3 \mu\text{m} \leq \lambda < 7 \mu\text{m} \\ \varepsilon_3 = 0.1, & 7 \mu\text{m} \leq \lambda < \infty \end{cases}$$

Determine the average emissivity of the surface and its emissive power.



**FIGURE 11-30**  
The spectral emissivity of the surface considered in Example 11-4.

**SOLUTION** The variation of emissivity of a surface at a specified temperature with wavelength is given. The average emissivity of the surface and its emissive power are to be determined.

**Analysis** The variation of the emissivity of the surface with wavelength is given as a step function. Therefore, the average emissivity of the surface can be determined from Eq. 11-34 by breaking the integral into three parts,

$$\begin{aligned}\varepsilon(T) &= \frac{\varepsilon_1 \int_0^{\lambda_1} E_{b\lambda} d\lambda}{\sigma T^4} + \frac{\varepsilon_2 \int_{\lambda_1}^{\lambda_2} E_{b\lambda} d\lambda}{\sigma T^4} + \frac{\varepsilon_3 \int_{\lambda_2}^{\infty} E_{b\lambda} d\lambda}{\sigma T^4} \\ &= \varepsilon_1 f_{0-\lambda_1}(T) + \varepsilon_2 f_{\lambda_1-\lambda_2}(T) + \varepsilon_3 f_{\lambda_2-\infty}(T) \\ &= \varepsilon_1 f_{\lambda_1} + \varepsilon_2 (f_{\lambda_2} - f_{\lambda_1}) + \varepsilon_3 (1 - f_{\lambda_2})\end{aligned}$$

where  $f_{\lambda_1}$  and  $f_{\lambda_2}$  are blackbody radiation functions corresponding to  $\lambda_1 T$  and  $\lambda_2 T$ . These functions are determined from Table 11-2 to be

$$\lambda_1 T = (3 \mu\text{m})(800 \text{ K}) = 2400 \mu\text{m} \cdot \text{K} \rightarrow f_{\lambda_1} = 0.140256$$

$$\lambda_2 T = (7 \mu\text{m})(800 \text{ K}) = 5600 \mu\text{m} \cdot \text{K} \rightarrow f_{\lambda_2} = 0.701046$$

Note that  $f_{0-\lambda_1} = f_{\lambda_1} - f_0 = f_{\lambda_1}$ , since  $f_0 = 0$ , and  $f_{\lambda_2-\infty} = f_{\infty} - f_{\lambda_2} = 1 - f_{\lambda_2}$ , since  $f_{\infty} = 1$ . Substituting,

$$\begin{aligned}\varepsilon &= 0.3 \times 0.140256 + 0.8(0.701046 - 0.140256) + 0.1(1 - 0.701046) \\ &= \mathbf{0.521}\end{aligned}$$

That is, the surface will emit as much radiation energy at 800 K as a gray surface having a constant emissivity of  $\varepsilon = 0.521$ . The emissive power of the surface is

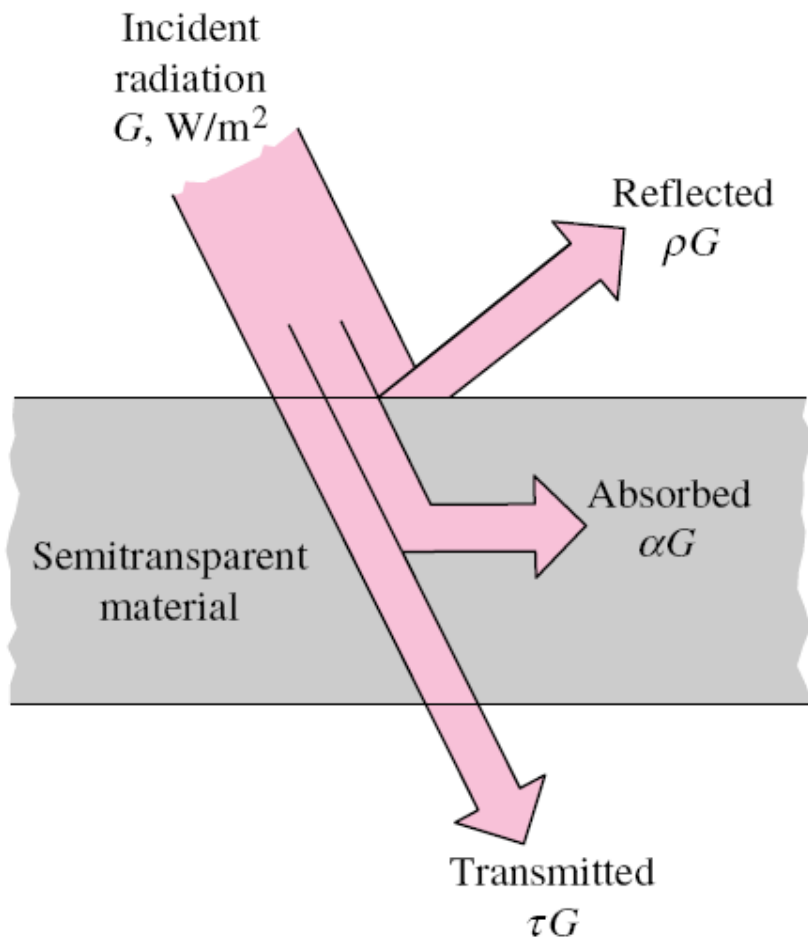
$$E = \varepsilon \sigma T^4 = 0.521(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)(800 \text{ K})^4 = \mathbf{12,100 \text{ W/m}^2}$$

*Absorptivity:*  $\alpha = \frac{\text{Absorbed radiation}}{\text{Incident radiation}} = \frac{G_{\text{abs}}}{G}, \quad 0 \leq \alpha \leq 1$

*Reflectivity:*  $\rho = \frac{\text{Reflected radiation}}{\text{Incident radiation}} = \frac{G_{\text{ref}}}{G}, \quad 0 \leq \rho \leq 1$

*Transmissivity:*  $\tau = \frac{\text{Transmitted radiation}}{\text{Incident radiation}} = \frac{G_{\text{tr}}}{G}, \quad 0 \leq \tau \leq 1$

**Irradiation,  $G$ :**  
Radiation flux  
incident on a  
surface.



## Absorptivity, Reflectivity, and Transmissivity

$$G_{\text{abs}} + G_{\text{ref}} + G_{\text{tr}} = G$$

$$\alpha + \rho + \tau = 1$$

$$\alpha + \rho = 1 \quad \text{for opaque surfaces}$$

**FIGURE 12-31**

The absorption, reflection, and transmission of incident radiation by a semitransparent material.



$$\alpha_\lambda(\lambda) = \frac{G_{\lambda, \text{abs}}(\lambda)}{G_\lambda(\lambda)}$$

spectral hemispherical absorptivity

spectral directional absorptivity

$$\rho_\lambda(\lambda) = \frac{G_{\lambda, \text{ref}}(\lambda)}{G_\lambda(\lambda)}$$

spectral hemispherical reflectivity

$$\alpha_{\lambda, \theta}(\lambda, \theta, \phi) = \frac{I_{\lambda, \text{abs}}(\lambda, \theta, \phi)}{I_{\lambda, i}(\lambda, \theta, \phi)}$$

spectral directional reflectivity

$$\tau_\lambda(\lambda) = \frac{G_{\lambda, \text{tr}}(\lambda)}{G_\lambda(\lambda)}$$

spectral hemispherical transmissivity

$$\rho_{\lambda, \theta}(\lambda, \theta, \phi) = \frac{I_{\lambda, \text{ref}}(\lambda, \theta, \phi)}{I_{\lambda, i}(\lambda, \theta, \phi)}$$

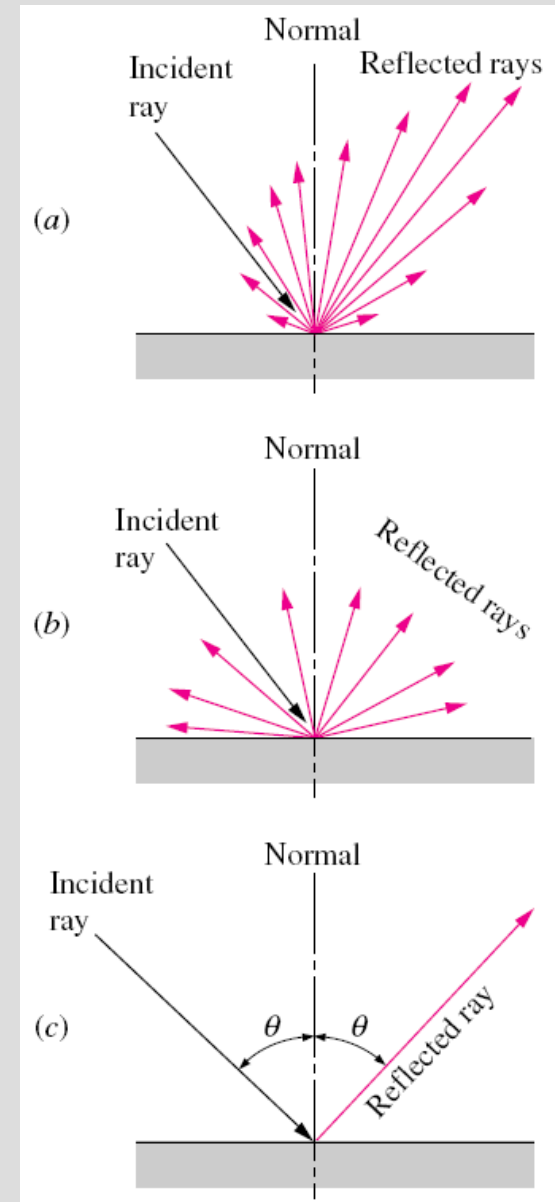
$G_\lambda$ : the spectral irradiation,  $\text{W}/\text{m}^2 \cdot \mu\text{m}$

Average absorptivity, reflectivity, and transmissivity of a surface:

$$\alpha = \frac{\int_0^\infty \alpha_\lambda G_\lambda d\lambda}{\int_0^\infty G_\lambda d\lambda}, \quad \rho = \frac{\int_0^\infty \rho_\lambda G_\lambda d\lambda}{\int_0^\infty G_\lambda d\lambda}, \quad \tau = \frac{\int_0^\infty \tau_\lambda G_\lambda d\lambda}{\int_0^\infty G_\lambda d\lambda}$$

**FIGURE 12–32**

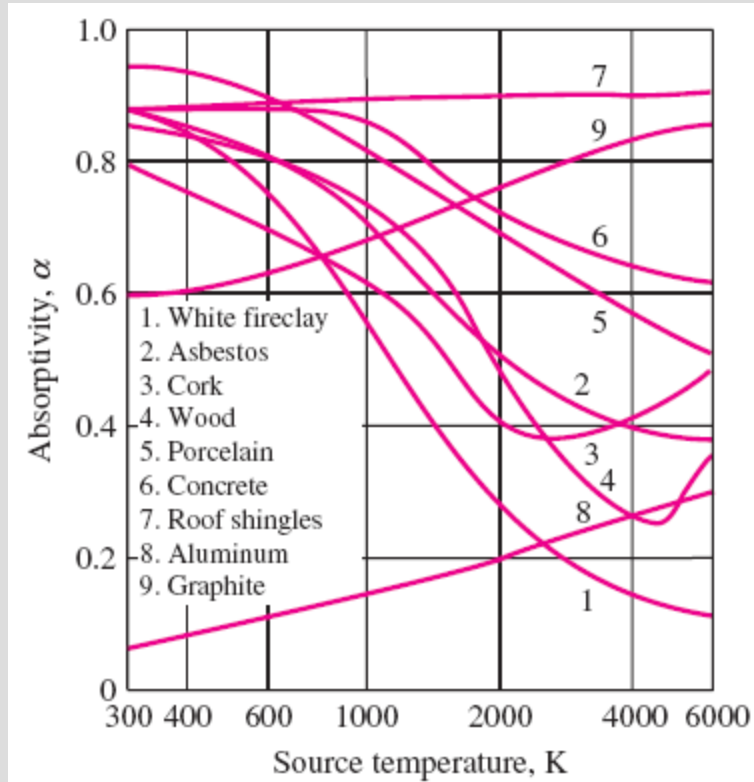
Different types of reflection from a surface: (a) actual or irregular, (b) diffuse, and (c) specular or mirrorlike.



In practice, surfaces are assumed to reflect in a perfectly *specular* or *diffuse* manner.

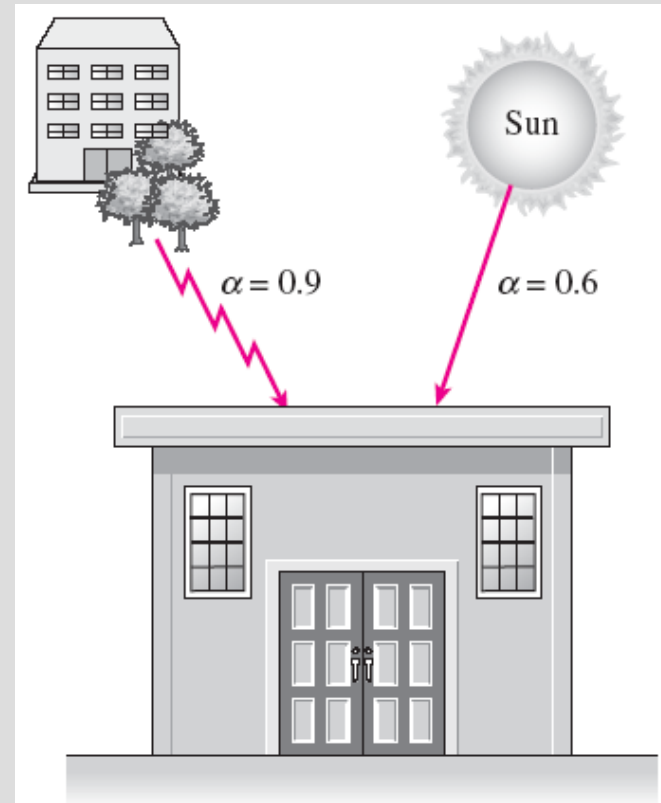
**Specular (or mirrorlike) reflection:** *The angle of reflection equals the angle of incidence of the radiation beam.*

**Diffuse reflection:** *Radiation is reflected equally in all directions.*



**FIGURE 12-33**

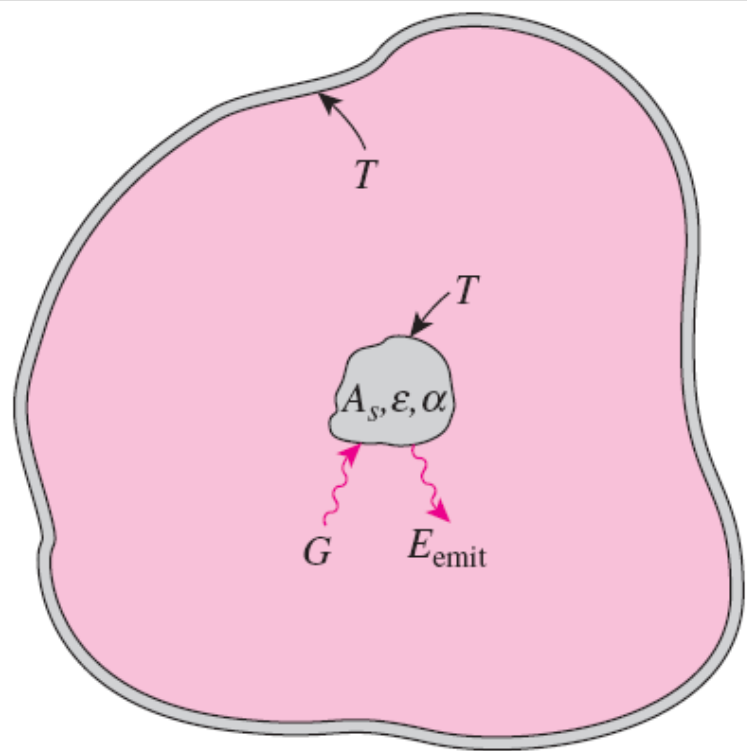
Variation of absorptivity with the temperature of the source of irradiation for various common materials at room temperature.



**FIGURE 12-34**

The absorptivity of a material may be quite different for radiation originating from sources at different temperatures.

# Kirchhoff's Law



**FIGURE 12-35**

The small body contained in a large isothermal enclosure used in the development of Kirchhoff's law.

$$\varepsilon_{\lambda, \theta}(T) = \alpha_{\lambda, \theta}(T).$$

The emissivity of a surface at a specified wavelength, direction, and temperature is always equal to its absorptivity at the same wavelength, direction, and temperature.

$$G_{\text{abs}} = \alpha G = \alpha \sigma T^4$$

$$E_{\text{emit}} = \varepsilon \sigma T^4$$

$$A_s \varepsilon \sigma T^4 = A_s \alpha \sigma T^4$$

$$\varepsilon(T) = \alpha(T) \quad \text{Kirchhoff's law}$$

*The total hemispherical emissivity of a surface at temperature T is equal to its total hemispherical absorptivity for radiation coming from a blackbody at the same temperature.*

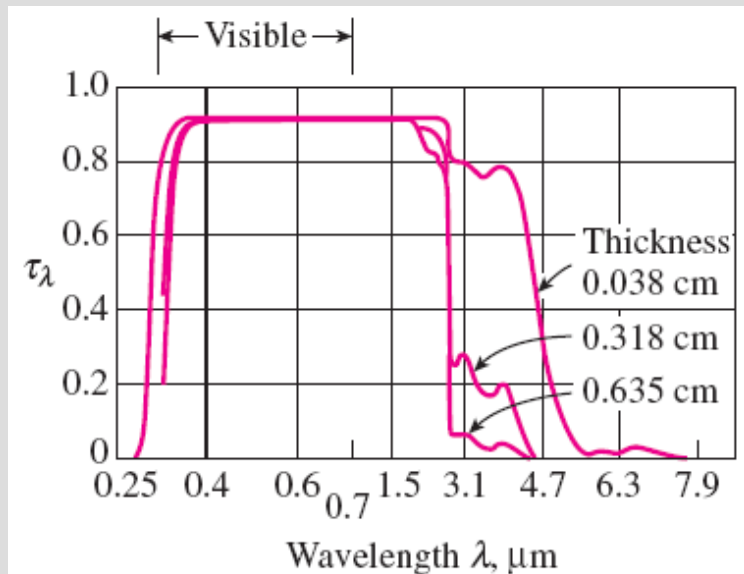
$$\varepsilon_{\lambda}(T) = \alpha_{\lambda}(T) \quad \text{spectral form of Kirchhoff's law}$$

# The Greenhouse Effect

Glass has a **transparent window** in the wavelength range  $0.3 \mu\text{m} < \lambda < 3 \mu\text{m}$  in which over 90% of solar radiation is emitted. The entire radiation emitted by surfaces at room temperature falls in the infrared region ( $\lambda > 3 \mu\text{m}$ ).

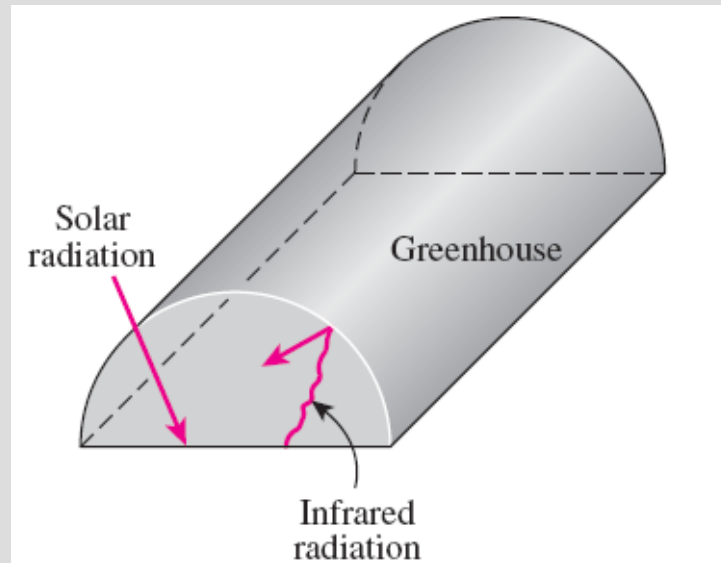
Glass allows the solar radiation to enter but does not allow the infrared radiation from the interior surfaces to escape. This causes a rise in the interior temperature as a result of the energy buildup in the car.

This heating effect, which is due to the nongray characteristic of glass (or clear plastics), is known as the **greenhouse effect**.



**FIGURE 12-36**

The spectral transmissivity of low-iron glass at room temperature for different thicknesses.



**FIGURE 12-37**

A greenhouse traps energy by allowing the solar radiation to come in but not allowing the infrared radiation to go out.

# ATMOSPHERIC AND SOLAR RADIATION

**Atmospheric radiation:** The radiation energy emitted or reflected by the constituents of the atmosphere.

The energy of the sun is due to the continuous *fusion* reaction during which two hydrogen atoms fuse to form one atom of helium.

Therefore, the sun is essentially a *nuclear reactor*, with temperatures as high as **40,000,000 K** in its core region.

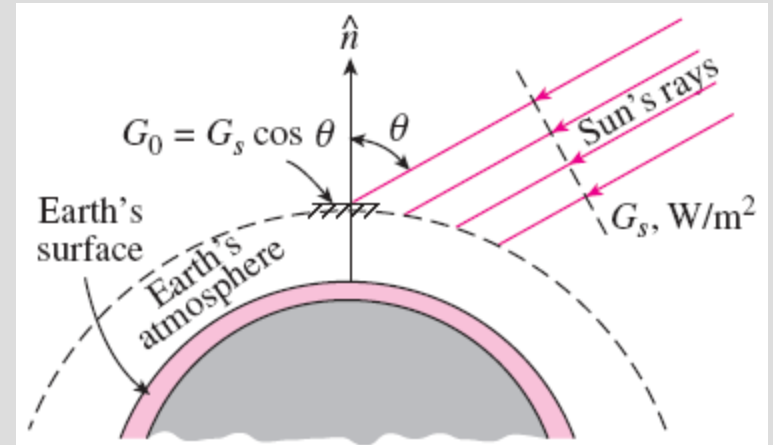
The temperature drops to about **5800 K** in the outer region of the sun, called the convective zone, as a result of the dissipation of this energy by radiation.

## Total solar irradiance $G_s$ :

The solar energy reaching the earth's atmosphere is called the

$$G_s = 1373 \text{ W/m}^2$$

**Solar constant:** The total solar irradiance. It represents *the rate at which solar energy is incident on a surface normal to the sun's rays at the outer edge of the atmosphere when the earth is at its mean distance from the sun*

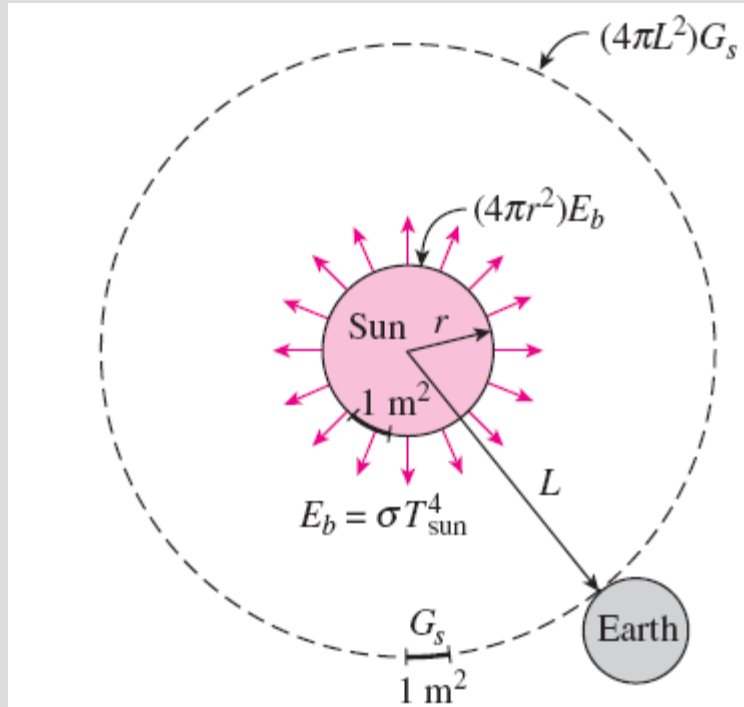


**FIGURE 12-38**

Solar radiation reaching the earth's atmosphere and the total solar irradiance.

The value of the total solar irradiance can be used to estimate the **effective surface temperature** of the sun from the requirement that

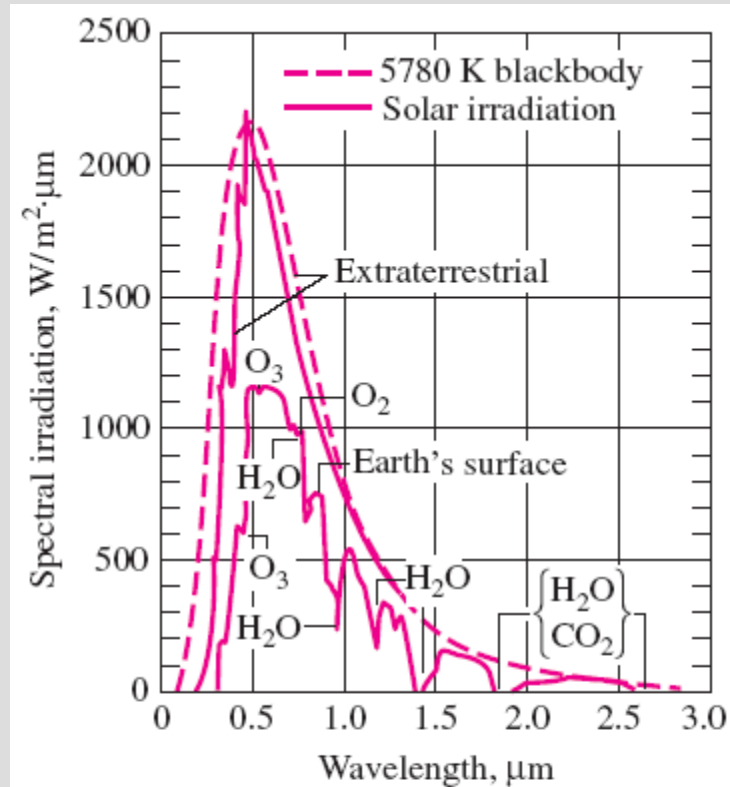
$$(4\pi L^2)G_s = (4\pi r^2)\sigma T_{\text{sun}}^4$$



**FIGURE 12–39**

The total solar energy passing through concentric spheres remains constant, but the energy falling per unit area decreases with increasing radius.

The sun can be treated as a blackbody at a temperature of 5780 K.

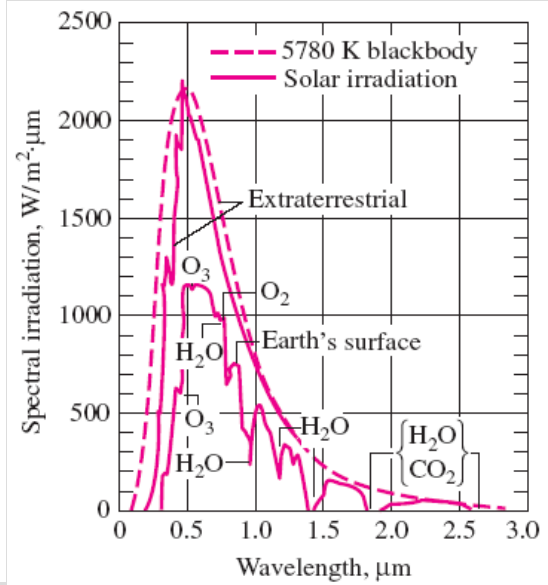


**FIGURE 12–40**

Spectral distribution of solar radiation just outside the atmosphere, at the surface of the earth on a typical day, and comparison with blackbody radiation at 5780 K.

The spectral distribution of solar radiation on the ground plotted in Fig. 12–40 shows that the solar radiation undergoes considerable *attenuation* as it passes through the atmosphere as a result of absorption and scattering. About 99 percent of the atmosphere is contained within a distance of 30 km from the earth’s surface. The several dips on the spectral distribution of radiation on the earth’s surface are due to *absorption* by the gases O<sub>2</sub>, O<sub>3</sub> (ozone), H<sub>2</sub>O, and CO<sub>2</sub>. Absorption by *oxygen* occurs in a narrow band about  $\lambda = 0.76 \mu\text{m}$ . The *ozone* absorbs *ultraviolet* radiation at wavelengths below 0.3  $\mu\text{m}$  almost completely, and radiation in the range 0.3–0.4  $\mu\text{m}$  considerably. Thus, the ozone layer in the upper regions of the atmosphere protects biological systems on earth from harmful ultraviolet radiation. In turn, we must protect the ozone layer from the destructive chemicals commonly used as refrigerants, cleaning agents, and propellants in aerosol cans. The use of these chemicals is now banned. The ozone gas also absorbs some radiation in the visible range. Absorption in the infrared region is dominated by *water vapor* and *carbon dioxide*. The dust particles and other pollutants in the atmosphere also absorb radiation at various wavelengths.

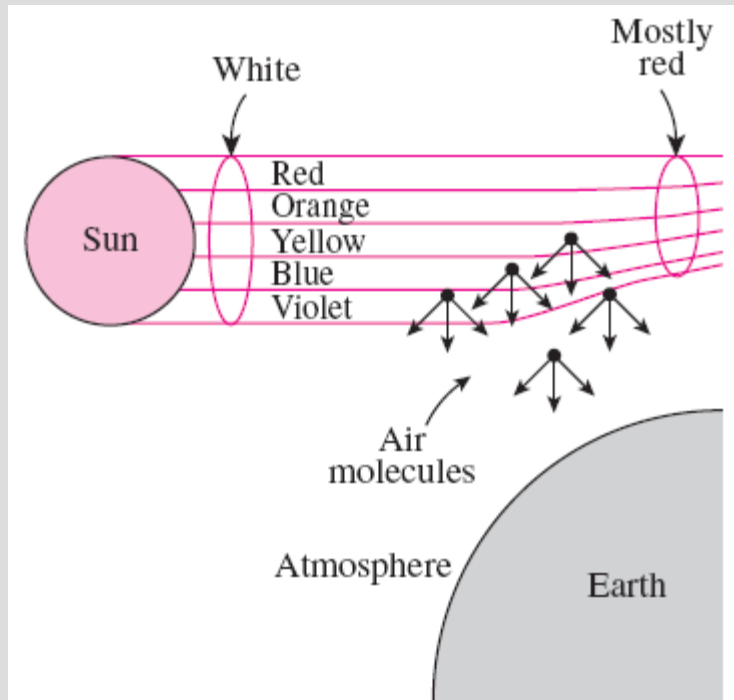
As a result of these absorptions, the solar energy reaching the earth’s surface is weakened considerably, to about 950 W/m<sup>2</sup> on a clear day and much less on cloudy or smoggy days. Also, practically all of the solar radiation reaching the earth’s surface falls in the wavelength band from 0.3 to 2.5  $\mu\text{m}$ .



**FIGURE 12–40** Spectral distribution of solar radiation just outside the atmosphere, at the surface of the earth on a typical day, and comparison with blackbody radiation at 5780 K.



The solar energy incident on a surface on earth is considered to consist of *direct* and *diffuse* parts.



**FIGURE 12-41**

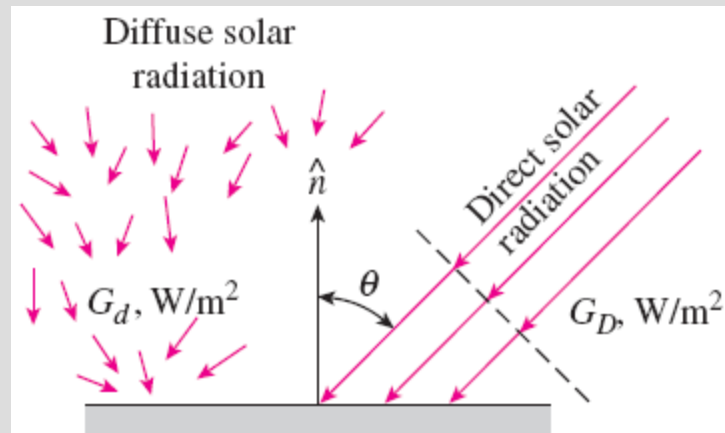
Air molecules scatter blue light much more than they do red light. At sunset, light travels through a thicker layer of atmosphere, which removes much of the blue from the natural light, allowing the red to dominate.

**Direct solar radiation  $G_D$ :** The part of solar radiation that reaches the earth's surface without being scattered or absorbed by the atmosphere.

**Diffuse solar radiation  $G_d$ :** The scattered radiation is assumed to reach the earth's surface uniformly from all directions.

The *total* solar energy incident on the unit area of a *horizontal surface* on the ground is

$$G_{\text{solar}} = G_D \cos \theta + G_d \quad (\text{W/m}^2)$$



**FIGURE 12-42**

The direct and diffuse radiation incident on a horizontal surface on earth's surface.



it is found convenient in radiation calculations to treat the atmosphere as a blackbody at some lower fictitious temperature that emits an equivalent amount of radiation energy.

This fictitious temperature is called the **effective sky temperature**  $T_{\text{sky}}$ .

The radiation emission from the atmosphere to the earth's surface is

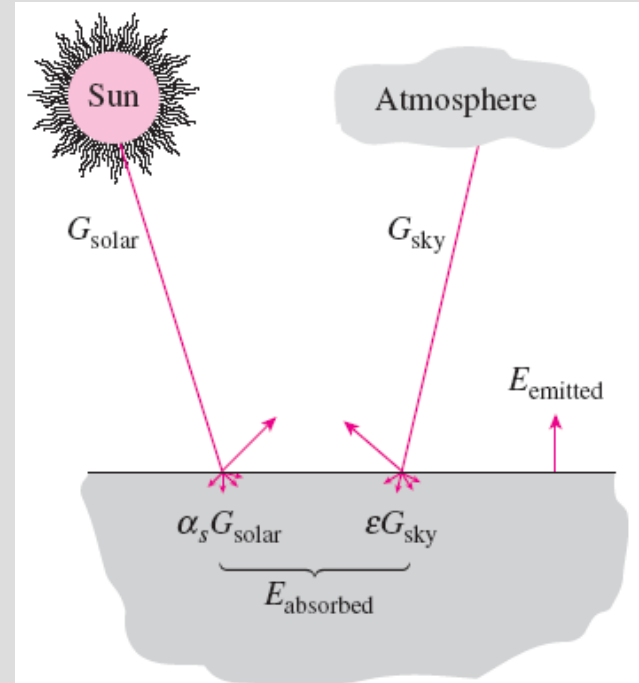
$$G_{\text{sky}} = \sigma T_{\text{sky}}^4 \quad (\text{W/m}^2)$$

The value of  $T_{\text{sky}}$  depends on the atmospheric conditions. It ranges from about 230 K for cold, clear-sky conditions to about 285 K for warm, cloudy-sky conditions.

$$E_{\text{sky, absorbed}} = \alpha G_{\text{sky}} = \alpha \sigma T_{\text{sky}}^4 = \varepsilon \sigma T_{\text{sky}}^4 \quad (\text{W/m}^2)$$

Net rate of radiation heat transfer to a surface exposed to solar and atmospheric radiation

$$\begin{aligned} \dot{q}_{\text{net, rad}} &= \sum E_{\text{absorbed}} - \sum E_{\text{emitted}} \\ &= E_{\text{solar, absorbed}} + E_{\text{sky, absorbed}} - E_{\text{emitted}} \\ &= \alpha_s G_{\text{solar}} + \varepsilon \sigma T_{\text{sky}}^4 - \varepsilon \sigma T_s^4 \\ &= \alpha_s G_{\text{solar}} + \varepsilon \sigma (T_{\text{sky}}^4 - T_s^4) \quad (\text{W/m}^2) \end{aligned}$$



**FIGURE 12-43**

Radiation interactions of a surface exposed to solar and atmospheric radiation.

The absorption and emission of radiation by the *elementary gases* such as H<sub>2</sub>, O<sub>2</sub>, and N<sub>2</sub> at moderate temperatures are negligible, and a medium filled with these gases can be treated as a *vacuum* in radiation analysis.

The absorption and emission of gases with *larger molecules* such as H<sub>2</sub>O and CO<sub>2</sub>, however, can be *significant* and may need to be considered when considerable amounts of such gases are present in a medium.

For example, a 1-m-thick layer of water vapor at 1 atm pressure and 100°C emits more than 50 percent of the energy that a blackbody would emit at the same temperature.

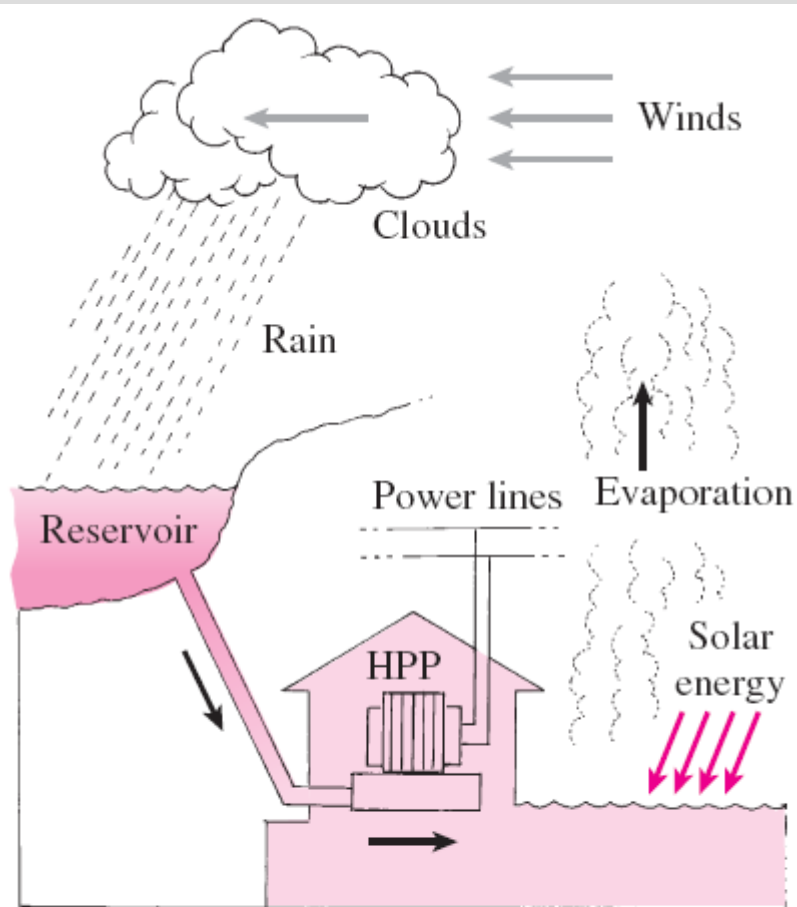
The radiation properties of surfaces are quite different for the incident and emitted radiation, and the surfaces cannot be assumed to be gray.

Instead, the surfaces are assumed to have two sets of properties: *one for solar radiation* and *another for infrared radiation at room temperature*.

**TABLE 12-3**

Comparison of the solar absorptivity  $\alpha_s$  of some surfaces with their emissivity  $\varepsilon$  at room temperature

Surface	$\alpha_s$	$\varepsilon$
Aluminum		
Polished	0.09	0.03
Anodized	0.14	0.84
Foil	0.15	0.05
Copper		
Polished	0.18	0.03
Tarnished	0.65	0.75
Stainless steel		
Polished	0.37	0.60
Dull	0.50	0.21
Plated metals		
Black nickel oxide	0.92	0.08
Black chrome	0.87	0.09
Concrete	0.60	0.88
White marble	0.46	0.95
Red brick	0.63	0.93
Asphalt	0.90	0.90
Black paint	0.97	0.97
White paint	0.14	0.93
Snow	0.28	0.97
Human skin (Caucasian)	0.62	0.97



**FIGURE 12-44**

The cycle that water undergoes in a hydroelectric power plant.

What we call *renewable energy* is usually nothing more than the manifestation of solar energy in different forms.

Such energy sources include *wind energy*, *hydroelectric power*, *ocean thermal energy*, *ocean wave energy*, and *wood*.

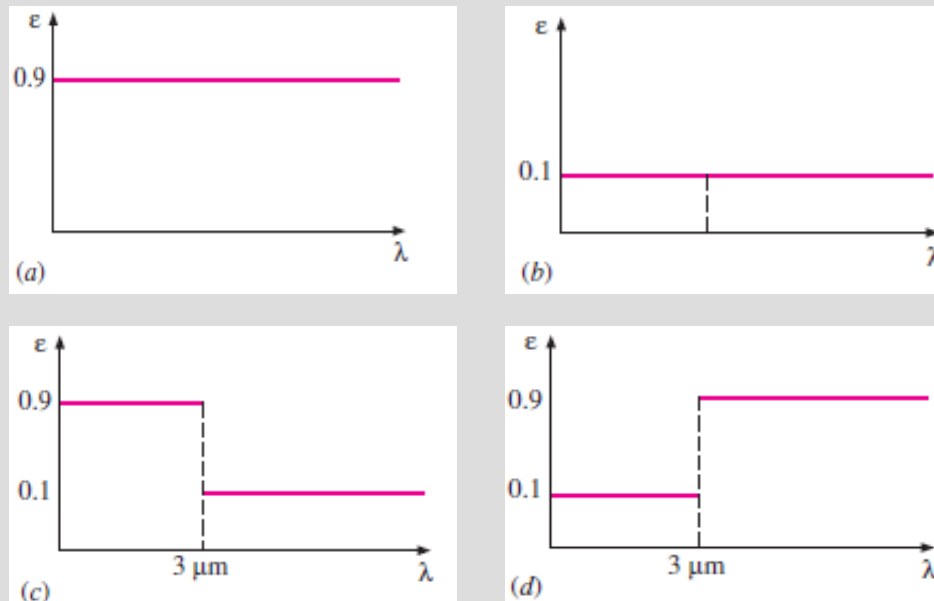
For example, no hydroelectric power plant can generate electricity year after year unless the water evaporates by absorbing solar energy and comes back as a rainfall to replenish the water source.

Although solar energy is sufficient to meet the entire energy needs of the world, currently it is *not economical* to do so because of the *low concentration of solar energy* on earth and the *high capital cost* of harnessing it.

## EXAMPLE 11–5 Selective Absorber and Reflective Surfaces

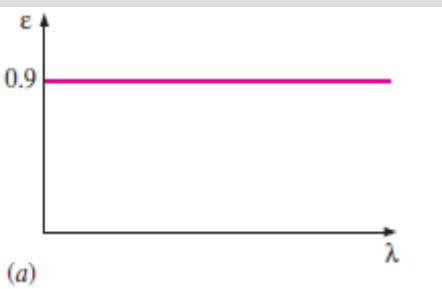
Consider a surface exposed to solar radiation. At a given time, the direct and diffuse components of solar radiation are  $G_D = 400$  and  $G_d = 300$  W/m<sup>2</sup>, and the direct radiation makes a 20° angle with the normal of the surface. The surface temperature is observed to be 320 K at that time. Assuming an effective sky temperature of 260 K, determine the net rate of radiation heat transfer for these cases (Fig. 11–45):

- (a)  $\alpha_s = 0.9$  and  $\varepsilon = 0.9$  (gray absorber surface)
- (b)  $\alpha_s = 0.1$  and  $\varepsilon = 0.1$  (gray reflector surface)
- (c)  $\alpha_s = 0.9$  and  $\varepsilon = 0.1$  (selective absorber surface)
- (d)  $\alpha_s = 0.1$  and  $\varepsilon = 0.9$  (selective reflector surface)



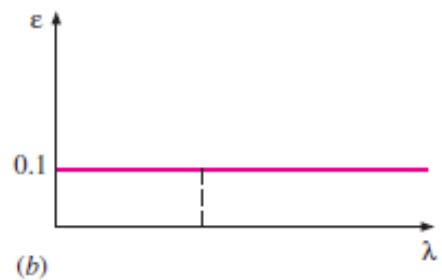
**FIGURE 11–45**

Graphical representation of the spectral emissivities of the four surfaces considered in Example 11–5.



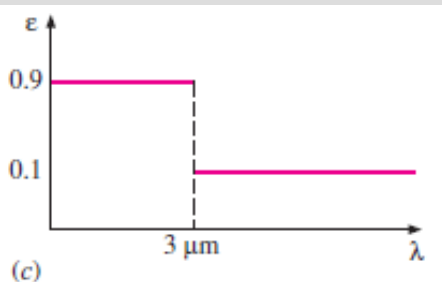
(a)  $\alpha_s = 0.9$  and  $\varepsilon = 0.9$  (gray absorber surface):

$$\begin{aligned}\dot{q}_{\text{net, rad}} &= 0.9(676 \text{ W/m}^2) + 0.9(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)[(260 \text{ K})^4 - (320 \text{ K})^4] \\ &= 307 \text{ W/m}^2\end{aligned}$$



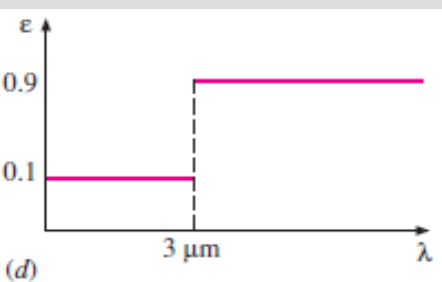
(b)  $\alpha_s = 0.1$  and  $\varepsilon = 0.1$  (gray reflector surface):

$$\begin{aligned}\dot{q}_{\text{net, rad}} &= 0.1(676 \text{ W/m}^2) + 0.1(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)[(260 \text{ K})^4 - (320 \text{ K})^4] \\ &= 34 \text{ W/m}^2\end{aligned}$$



(c)  $\alpha_s = 0.9$  and  $\varepsilon = 0.1$  (selective absorber surface):

$$\begin{aligned}\dot{q}_{\text{net, rad}} &= 0.9(676 \text{ W/m}^2) + 0.1(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)[(260 \text{ K})^4 - (320 \text{ K})^4] \\ &= 575 \text{ W/m}^2\end{aligned}$$



(d)  $\alpha_s = 0.1$  and  $\varepsilon = 0.9$  (selective reflector surface):

$$\begin{aligned}\dot{q}_{\text{net, rad}} &= 0.1(676 \text{ W/m}^2) + 0.9(5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4)[(260 \text{ K})^4 - (320 \text{ K})^4] \\ &= -234 \text{ W/m}^2\end{aligned}$$

# Summary

- Introduction
- Thermal Radiation
- Blackbody Radiation
- Radiation Intensity
  - ✓ Solid Angle
  - ✓ Intensity of Emitted Radiation
  - ✓ Incident Radiation
  - ✓ Radiosity
  - ✓ Spectral Quantities
- Radiative Properties
  - ✓ Emissivity
  - ✓ Absorptivity, Reflectivity, and Transmissivity
  - ✓ Kirchhoff's Law
  - ✓ The Greenhouse Effect
- Atmospheric and Solar Radiation